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A General-Purpose Preconditioner for Method of Moments Matrices and a Novel Approach to Resolving the Low Frequency Breakdown Problem

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Abstract

This paper presents a general-purpose preconditioner for Method of Moments (MoM) matrices to speed up iterative solutions of the same. The proposed preconditioner is constructed by inverting a sparsified version of the impedance matrix, which is found to retain all the important singular values of the original impedance matrix. The preconditioner is shown to significantly reduce the number of iterations while using an iterative solver such as Generalized Minimal Residual (GMRES). We test the efficacy of the preconditioner for a variety of ill-conditioned MoM problems, including those arising from internal resonance and non-uniform meshing of multiscale problems. Additionally, we show that a further speedup of the convergence can be achieved, without compromising the accuracy, by using an alternative convergence criterion. Finally, the problem of ill-conditioning arising from the low-frequency breakdown problem is also examined, and a novel strategy for handling such problems is proposed as an alternative to using the loop-star basis function.

1. Introduction

Dealing with ill-conditioned matrices is one of the most critical issues, frequently encountered when using the Method of Moments (MoM) for EM modeling problems [1]. Such ill-conditioning may arise due to a variety of reasons, such as the internal resonance, low-frequency breakdown, and the use of poorly conditioned mesh for a multiscale geometry. A number of preconditioners, including those based on the Singular Value Decomposition (SVD), have been proposed in the literature [2]–[4] to handle this problem. However, constructing a general-purpose preconditioner, which is easy to implement in a cost-effective manner, is still an active area of research.

Recently, a robust pre-conditioning technique, which is based on a singular value filtering approach of the MoM matrix, has been presented in [5], [6]. There it is shown that we can significantly improve the performance of an iterative solvers, e.g. the Generalized Minimal Residual (GMRES), without compromising the accuracy of the results, such as the RADAR Cross Section (RCS), by filtering out the last few singular values that are responsible for making the system ill-conditioned. However, one needs to obtain the SVD of a large impedance matrix, which is computationally as expensive as solving the linear system of equations, in the algorithm introduced in [5], [6]. In this paper, we introduce an interesting technique for bypassing the computation of SVD in the next section.

2. Proposed Pre-conditioning Technique

We introduce an efficient way of constructing an easy-to-invert matrix, whose singular values are very similar to those of the original impedance matrix. Towards this end, we begin with the MoM equation:

$$ZI = V, \tag{1}$$

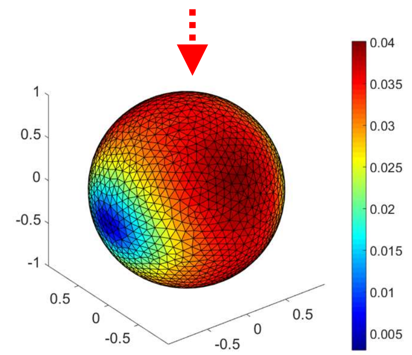
where Z is the impedance matrix, I is the unknown coefficient vector, and V is the excitation vector. We apply the proposed preconditioning technique to two very different types of test problems. The first of these is associated with the problem of scattering by a Perfect Electric Conductor (PEC) sphere at its frequency of internal resonance (see Figs. 1–6). The second problem is that of a multi-scale structure, viz., a PEC cone (see Figs. 7–10) with a narrow tip.

Our first step is to construct a thresholded impedance matrix Z_{Th}, by setting all the entries of Z that are lower than a threshold value, which we pre-define, to zero. As a result, the matrix Z_{Th} becomes highly sparse matrix with typical retention factors of only 4% to 6%, as may be seen from Fig. 1 (b) for the test problem of a PEC sphere. Next, we construct the preconditioner P such that $P = Z_{Th}^{-1}$ in a cost-effective manner, since the inversion of Z_{Th} is much less expensive than that of the original dense matrix Z. Finally, (1) is transformed into a well-conditioned system:

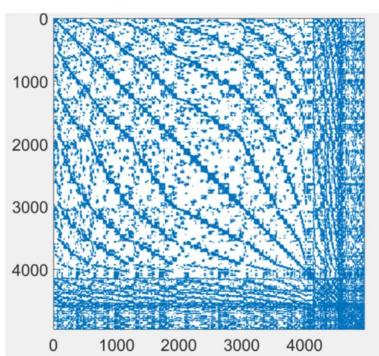
$$(PZ)I = PV. \tag{2}$$

Figs. 2 and 8 show that Z_{Th} inherits essentially all of the significant singular values from Z and thereby, $P = Z_{Th}^{-1}$ becomes a good approximation of Z^{-1}. Fig. 3 clearly shows that PZ has much more clustered singular values around 8×10^{-1}. Such clustering of singular values, lowers the Frobenius norm $\|I - PZ\|_F$, where I is the identity matrix, resulting in substantial improvement in the

GMRES convergence (see Figs. 4 and 9) [2]. Figs. 6 and 10 show that the accuracy of RCS is not compromised, when we apply the proposed preconditioner.



(a)



(b)

Figure 1. (a) The surface current induced on a PEC sphere of radius of 1 m at the internal resonance matrix of 131.55 MHz; (b) The sparsity pattern of $\mathbf{4959 \times 4959}$ $\boldsymbol{Z_{Th}}$, where the retention percentage is only $\mathbf{4.25\%}$. Here the threshold level is $\mathbf{10^{-3}}$.

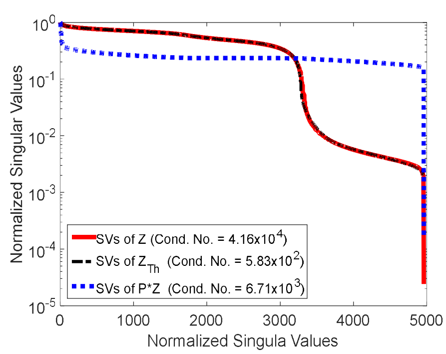


Figure 2. Normalized singular value distribution of the $\boldsymbol{Z}$, Z_{Th}, and $\boldsymbol{P} * \boldsymbol{Z}$ of the PEC sphere internal resonance problem. It is observed that the singular values of $\boldsymbol{Z}$ and Z_{Th} are the same except last few.

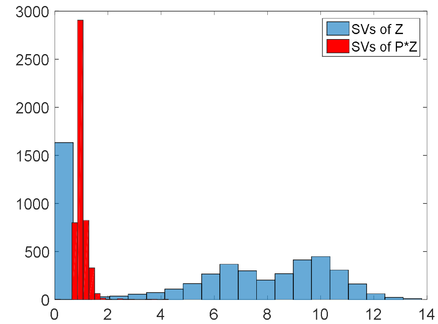


Figure 3. Histogram of singular values of the original and the preconditioned impedance matrix of the PEC sphere internal resonance problem.

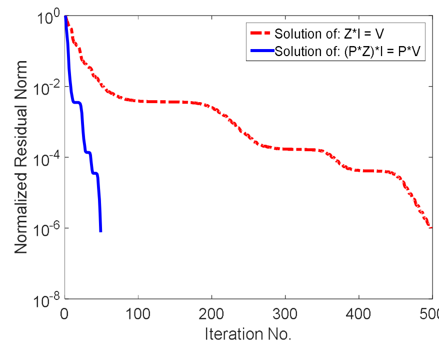


Figure 4. GMRES convergence for the original and preconditioned system of linear equations in the case of PEC sphere internal resonance problem. Evidently, the proposed preconditioner reduces the number of iterations from 498 to 49.

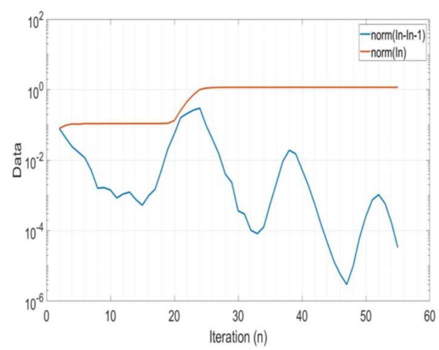


Figure 5. An alternative convergence criterion for GMRES applied to the case of the PEC sphere internal resonance problem.

Apart from introducing the proposed preconditioner, we present a macro-basis-function-based approach as an alternative to the standard loop-star basis function to handle the low-frequency breakdown problem of electric–field–integral–equation (EFIE). *This macro basis function technique*, which allows us to solve the problem with much fewer iterations at a considerably higher frequency, where condition number is comparatively lower, *is proposed based upon the crucial observation that the nature of both the distribution of surface current density and RCS do not*

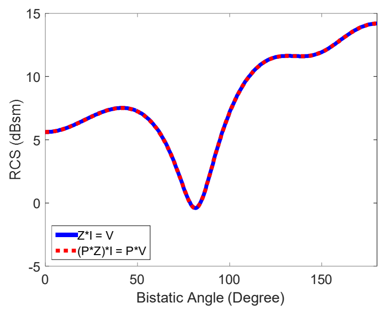


Figure 6. The RCS of the PEC sphere, obtained by solving the original and the preconditioned system of linear equations.

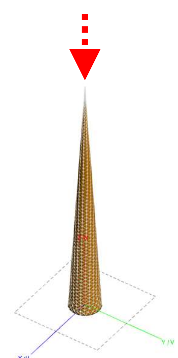


Figure 7. Geometry of the PEC cone scatterer with multiscale mesh, where the ratio of the longest to the shortest mesh edge is 228.5.

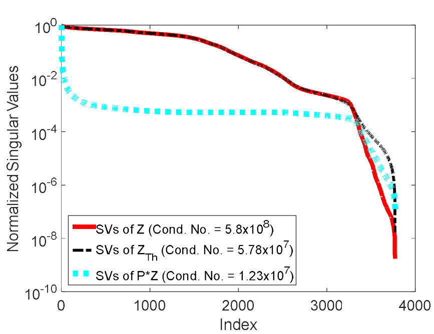


Figure 8. Normalized singular value distribution of the $\boldsymbol{Z}$, $\boldsymbol{Z_{Th}}$, and $\boldsymbol{P} * \boldsymbol{Z}$ for PEC cone scattering at 300 MHz. Most of the singular values of $\boldsymbol{P} * \boldsymbol{Z}$ tend to group around $\mathbf{5 \times 10^{-4}}$.

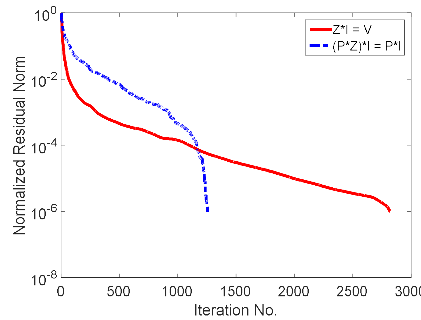


Figure 9. GMRES convergence for the original and preconditioned linear system in the case of PEC cone. Evidently, the proposed preconditioner reduces the number of iterations from 2814 to 1254.

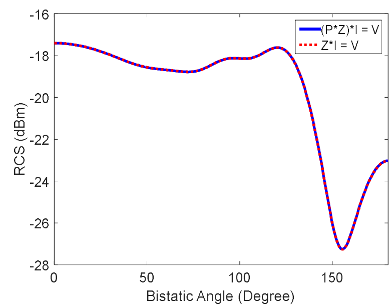


Figure 10. The RCS of the PEC cone, obtained by solving the original and preconditioned system of equations.

change except multiplication of the same with certain scaling factor, which we solve as the unknown coefficient of the macro basis function, in the low-frequency breakdown region.

Finally, we introduce an alternative convergence criterion, where we use ℓ_2–norm of the surface current density instead of using the conventional residual norm based convergence criterion, for terminating the GMRES iteration sooner. Fig. 5 shows that we can reduce the number of GMRES iterations from 498 to only 25, for instance in the case of internal resonance problem of PEC sphere, without compromising the accuracy of the RCS results.

3. References

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