On the Mean Effective Gain Expressed in Terms of the Spherical Vector Wave Expansion of the Electromagnetic Field

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Abstract

The mode expansion offers a general framework for the analysis of the interaction between antennas and propagation channels. In this paper, the Mean Effective Gain (MEG) of an antenna is expressed in terms of the spherical vector wave expansion of the electromagnetic field. An explicit expression of the MEG is provided as a function of the normalized average power of modes excited in the propagation channel and the correlation between the channel modes due to the polarization and spatial selectivity of plane waves impinging at the antenna.

1 Introduction

The mean effective gain (MEG) is a figure of merit of antennas that characterizes the interaction between arbitrary antennas and channels [1]. The MEG expression presented in this paper provides a generalized and systematic approach to the analysis of this interaction. The properties of both the antenna and the channel are represented in terms of the spherical vector wave multi-pole expansion of the electromagnetic field. This expansion gives a condensed interpretation of the antenna radiation properties summarized in the antenna scattering matrix, [2]. The advantages of such a formulation over the classical MEG expression are manifold: 1) it provides a straightforward physical interpretation of the interaction between the antenna and the propagation channel, 2) it enables a direct assessment of the multi-pole modes to be excited by the antenna in order to maximize the link gain, 3) the radiated performance of the antenna, or any equipment that uses an antenna for communication can be straightforwardly evaluated, 4) given a propagation environment, the best antenna could be tailored for the specific application with joint electromagnetic design and signal processing. Here we provide, based on the theory provided in [3], an explicit expression of the MEG as a function of the normalized average power of modes excited in the propagation channel and the correlation between the channel modes caused by the polarization and the spatial selectivity of plane waves impinging at the antenna.

2 Antenna Scattering Matrix

The scattering matrix contains all the properties of an arbitrary antenna with N ports as a transmitting, receiving or scattering device, [2],

$$\begin{pmatrix} \mathbf{\Gamma} & \mathbf{R} \\ \mathbf{T} & \mathbf{S} \end{pmatrix} \begin{pmatrix} \boldsymbol{v} \\ \boldsymbol{a} \end{pmatrix} = \begin{pmatrix} \boldsymbol{w} \\ \boldsymbol{b} \end{pmatrix}, \tag{1}$$

where the incoming waves, $\boldsymbol{a} \in \mathbb{C}^{M\times 1}$, the outgoing waves, $\boldsymbol{b} \in \mathbb{C}^{M\times 1}$, the incoming signals, $\boldsymbol{v} \in \mathbb{C}^{N\times 1}$ and the outgoing signals, $\boldsymbol{w} \in \mathbb{C}^{N\times 1}$ are related to each other through $\mathbf{\Gamma} \in \mathbb{C}^{N\times N}$, the matrix containing the complex antenna reflection coefficients, $\mathbf{R} \in \mathbb{C}^{N\times M}$, the matrix containing the antenna receiving coefficients, $\mathbf{T} \in \mathbb{C}^{M\times N}$ the matrix containing the antenna transmitting coefficients and $\mathbf{S} \in \mathbb{C}^{M\times M}$, the matrix containing the antenna scattering coefficients, where M is the maximum number of modes excited by the antenna, taken to be finite for all practical purposes.

The incoming waves, $\boldsymbol{a} = \{a_{\tau ml}\}$, and the outgoing waves, $\boldsymbol{b} = \{b_{\tau ml}\}$ are $M \times 1$ vectors of the multi-pole coefficients, obtained from the expansion of the total electric field $\boldsymbol{E}(\boldsymbol{r})$, in *incoming* $\boldsymbol{u}^{(1)}_{\tau ml}(k\boldsymbol{r})$ and *outgoing*

$\boldsymbol{u}^{(2)}_{\tau ml}(k\boldsymbol{r})$ spherical vector waves for $|\boldsymbol{r}| > a$,

$$\boldsymbol{E}(\boldsymbol{r}) = k\sqrt{2\eta}\sum_{l=1}^{l_{\max}}\sum_{m=-l}^{l}\sum_{\tau=1}^{2} a_{\tau ml}\boldsymbol{u}^{(1)}_{\tau ml}(k\boldsymbol{r}) + b_{\tau ml}\boldsymbol{u}^{(2)}_{\tau ml}(k\boldsymbol{r}), \tag{2}$$

where $a_{\tau ml}$ and $b_{\tau ml}$ are the corresponding multi-pole coefficients, $k = 2\pi/\lambda$ is the wave-number, λ is the wavelength, η is the free space impedance and $\boldsymbol{r}$ is the observation point. The multi-poles are classified as either TE ($\tau = 1$) or TM ($\tau = 2$). The azimuthal and radial dependencies are given by the m and l index, respectively. If $l_{\max}$ is the largest l-index considered then there are totally, $M = 2l_{\max}(l_{\max} + 1)$ modes. Whenever necessary, the multi–index ι is introduced and identified with the number $\iota = 2(l^2 + l - 1 + m) + \tau$. The normalization with $k\sqrt{2\eta}$ is used to give a power normalization of the expansion coefficients.

3 Mean Effective Gain

Consider a lossless, impedance matched antenna with N local ports. Assume then that the antenna operates as a receiver and that no outgoing waves are present, $\boldsymbol{b} = \boldsymbol{0}$. We can now establish a relationship between $\boldsymbol{w}$, the outgoing signals from the antenna ports and $\boldsymbol{a}$, the incoming waves impinging at the antenna, $\boldsymbol{w} = \mathbf{R}\boldsymbol{a}$. In [3] we define the MEG of an antenna to be the ratio of the average power of the outgoing signals, $\langle\|\boldsymbol{w}\|_F^2\rangle$, to the average power of the incoming waves, $\langle\|\boldsymbol{a}\|_F^2\rangle$. Equivalently, MEG is expressed as the ratio of $\mathrm{tr}\{\mathbf{R}_w\} = \mathrm{tr}\left\{\langle\boldsymbol{w}\boldsymbol{w}^\dagger\rangle\right\}$ and $\mathrm{tr}\{\mathbf{R}_a\} = \mathrm{tr}\left\{\langle\boldsymbol{a}\boldsymbol{a}^\dagger\rangle\right\}$,

$$G_{\mathrm{e}} = \frac{\mathrm{tr}\{\mathbf{R}_w\}}{\mathrm{tr}\{\mathbf{R}_a\}}. \tag{3}$$

Using results provided in [3], we arrive at the MEG expression in terms of the spherical vector wave multi-pole expansion,

$$G_{\mathrm{e}} = \sum_{\iota=1}^{M}|R_{n,\iota}|^2 P_{\tau ml} + 2\sum_{\iota}\sum_{\iota',\iota'\neq\iota}\Re\left\{R_{n,\iota}R^*_{n,\iota'}\rho_{\iota\iota'}\right\}, \tag{4}$$

where $\Re$ denotes real part, the normalized powers of the multi-pole modes excited by the antenna are given by the squared absolute value of the antenna reception coefficients, $|R_{n,\tau ml}|^2$, which satisfy the normalization: $\sum_{\tau=1}^{2}\sum_{m=-l}^{l}\sum_{l=1}^{l_{\max}}|R_{n,\tau ml}|^2 = \sum_{\iota=1}^{M}|R_{n,\iota}|^2 = 1$. P_ι is the normalized average power of modes excited in the propagation channel,

$$P_\iota = 4\pi\int\frac{\chi\,|A_{\iota,\theta}(\Omega)|^2\,p_\theta(\Omega) + |A_{\iota,\phi}(\Omega)|^2\,p_\phi(\Omega)}{\chi+1}\mathrm{d}\Omega, \tag{5}$$

and $\rho_{\iota\iota'}$ is the correlation between the channel modes due to the polarization and statistical (spatial) distribution of the AoA,

$$\rho_{\iota\iota'} = 4\pi(-\mathrm{i})^{l-l'-\tau+\tau'}\int\left(\frac{\chi p_\theta(\Omega)\,A^*_{\iota,\theta}(\Omega)\,A_{\iota',\theta}(\Omega) + p_\phi(\Omega)\,A^*_{\iota,\phi}(\Omega)\,A_{\iota',\phi}(\Omega)}{\chi+1}\right)\mathrm{d}\Omega, \tag{6}$$

where $p_\theta(\Omega)$ and $p_\phi(\Omega)$ are the probability density distributions (pdfs) of the angle of arrival (AoA) of the $\theta-$ and $\phi-$polarized components, respectively. The cross-polarization ratio χ, is defined as the ratio of the powers available in the $\theta-$ and $\phi-$polarizations, respectively and the functions $A_{\tau ml\theta}(\Omega)$ and $A_{\tau ml\phi}(\Omega)$ are the $\theta-$ and $\phi-$components of the spherical vector harmonics.

Furthermore, the partial gains of the antenna can be expressed as,

$$G_\alpha(\theta,\phi) = \sum_{\iota=1}^{M}|R_{n,\iota}|^2\,|A_{\iota,\alpha}(\Omega)|^2 + 2\sum_{\iota}\sum_{\iota',\iota'\neq\iota}\Re\left\{R_{n,\iota}R^*_{n,\iota'}A_{\iota,\alpha}(\Omega)\,A^*_{\iota',\alpha}(\Omega)\right\}, \tag{7}$$

where α takes on θ or ϕ. We have shown that the resulting MEG expression and the classical MEG equation in [1] are identical. However, the physical meaning of the MEG as expressed by Eq.(4) becomes clearer:

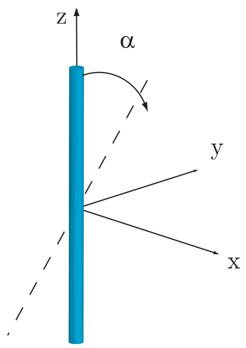

Figure 1: A $\pm 45°$ slanted polarization diversity arrangement. The rotation is towards the y-axis.

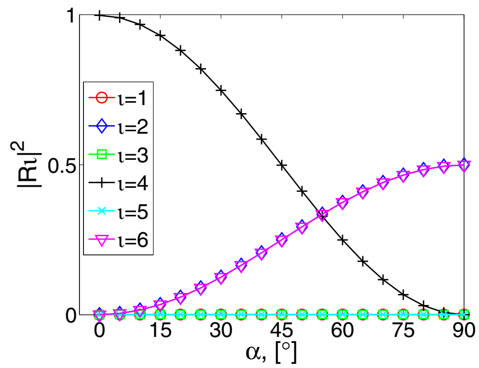

Figure 2: The squared absolute values of the reception (or transmission) coefficients, $|R_\iota|^2$, of the $\lambda/2$-dipole antenna as function of the rotation angle α and the ordered multi-pole multi-index $\iota = \{\tau, m, l\}$.

the MEG of an antenna is the normalized measure of the power coupling between the modes excited in the propagation channel and the modes excited in the antenna. We show in [3] that MEG is maximized when the mode coefficients of the antenna field are conjugate-matched to the modes of the propagation channel field.

4 Simulation results

Consider a lossless half-wavelength dipole antenna oriented along the z-axis. The dipole is further rotated around the x-axis, towards the positive y-axis as depicted in Fig. 1. Fig. 2 shows the squared absolute value of the receiver coefficient of the half-wavelength dipole as function of the tilting angle α. The rotation angle is denoted by $\alpha \in \left[0, \frac{\pi}{2}\right]$.

The pdfs of the AoA are assumed to be the same for both orthogonal polarizations and factorizable into azimuth and elevation, $p_\theta(\theta, \phi) = p_\phi(\theta, \phi) = p_\theta(\theta)\, p_\phi(\phi)$. The Laplacian pdf is then used to model the AoA, $p_\theta(\theta) = A_\theta \exp\left(-\sqrt{2}\,|\theta - \mu_\theta|/\sigma_\theta\right)\sin\theta$ and $p_\phi(\phi) = A_\phi \exp\left(\sqrt{2}\,|\phi - \mu_\phi|/\sigma_\phi\right)$, with parameters $\mu_\theta = \frac{\pi}{2}$, $\mu_\phi = 0$ and $\sigma_\theta = \sigma_\phi = \sigma$. As shown in Fig. 2 the power of the electric dipole modes (TM with $\iota = 2$, $\iota = 4$ and $\iota = 6$) of the half-wavelength antenna change as it is tilted. In the vertical position the power of the vertical electric dipole dominates, which has multi-index $\iota = 4$ and index modes $l = 1$, $\tau = 2$ and $m = 0$. However, the powers of the two "horizontal" electric dipoles ($\iota = 2$ and $\iota = 6$) increase as the tilt angle increases, while the power of the vertical electric dipole mode decreases. Observe that the powers of the two "horizontal" electric dipoles are always the same for half-wavelength antenna.

Fig. 3 shows the average power of the six lowest modes, (the dipole modes with $\iota = 1 \ldots 6$) as a function of the angle spread σ, as it varies from low (0.1rad) to very high (10rad), for three different values of the channel cross-polarization ratio, χ, -10 dB, 0 dB and 10 dB, respectively. Several observations can be made from Fig.3: 1) the channel XPR has a great impact on the power excited in the modes, 2) its impact is actually larger than the impact of the angle-spread, 3) the "vertical" electric dipole (TM with $\iota = 4$) and the "horizontal" magnetic dipole (TE with $\iota = 3$) modes are the most sensitives to both χ and σ, 4) the powers in the two "horizontal" electric dipoles (TM with $\iota = 2$ and $\iota = 6$) are always equal 5) the powers in the two "vertical" magnetic dipoles (TE with $\iota = 1$ and $\iota = 5$) are always equal. We can therefore conclude that for narrow band systems and when spatial interference is of no concern the link gain can be maximized by exciting modes with $\iota = 1, 4, 5$ for high χ, or modes $\iota = 2, 3, 6$ for low χ, while if χ is balanced all the six modes should be excited.

In Fig. 4 the MEG of the half-wavelength dipole is shown as a function of the tilt angle for three values

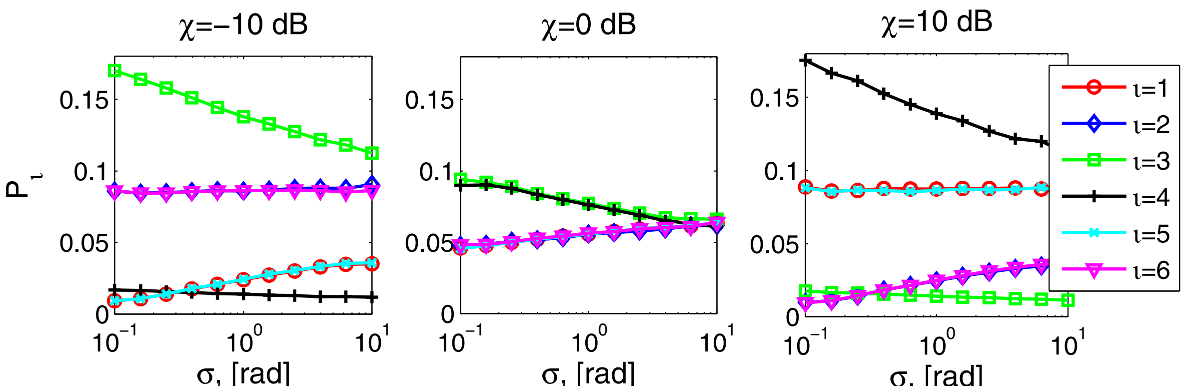

Figure 3: Average power of the six lowest modes, i.e., the dipole modes with $\iota = 1 \ldots 6$ as a function of the angle spread σ for three different χ.

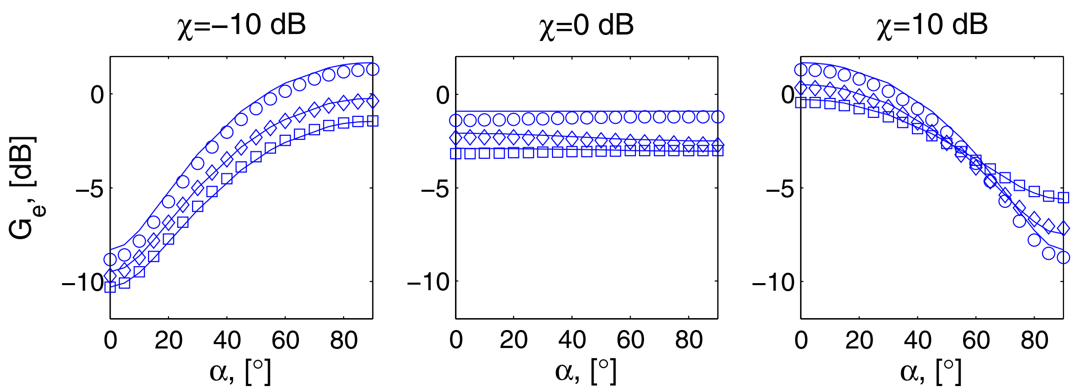

Figure 4: MEG of the $\lambda/2$-dipole is shown as a function of the tilt angle α. Markers '∘", "⋄" and "□" denote $\sigma = 0.1$ rad, $\sigma = 1$ rad, $\sigma = 10$ rad,respectively

of σ and three values of χ. Simulation results are shown with markers, while the MEG computed by the classical MEG expression, [1], is shown with a continuous line. As we can see the agreements are very good with small discrepancy of less than 0.3dB in average. As can be seen the MEG strongly depends upon the tilt angle, α, and the channel XPR, χ, however, it is considerably less sensitive to the angle-spread, σ.

Acknowledgements

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