Branch Cross-Correlation in Presence of Spatially Selective Interference Expressed in Terms of the Spherical Vector Wave Expansion of the Electromagnetic Field

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Abstract

In this paper we present an analysis of the cross-correlation coefficient between signals at two antenna branches (ports) in the presence of spatially selective interference and additive white gaussian noise. More specifically, we look at a $\pm 45^\circ$ slanted polarization diversity arrangement, which is rotated around the axis perpendicular to the plane containing the antennas. Results are provided as a function of the rotation angle. The analysis is based on the spherical vector wave multi-pole expansion of both the field impinging on the antennas and the antenna radiation properties.

1 Introduction

The correlation coefficient of the fading at available diversity branches is a vital characteristic of the polarization and spatial diversity inherent to multiple-antennas systems, [1], [2], [3], [4]. We show in this paper that it can be naturally expressed in terms of the properties of both antennas and channels by means of the spherical vector wave expansion of the electromagnetic field, [5]. This expansion gives a condensed interpretation of the antenna radiation properties summarized in the antenna scattering matrix, [6]. This approach is general; however, it is best suited for electrically small antennas or dipole-like antennas for which the six lowest modes of the electromagnetic field are dominating due to the high losses and low bandwidth associated with high order modes, [7]. Furthermore we recently showed that it is also highly suitable for describing the propagation channel and especially the interaction between antennas and propagation channels [8], which in turn governs the correlation coefficient. Several insights can be gained from this representation: 1) the physical meaning of the mechanisms that give rise to the correlation between the antenna branches, e.g., the correlation or coupling between the electromagnetic field multi-poles becomes apparent, 2) the expressions can be straightforwardly applied to quantify the impact of antenna cross-correlation on different antenna diversity schemes, and 3), it provides the necessary tools to study the branch-correlation in the presence of spatially selective interference. Results presented here are based on the theory outlined in [8].

2 Correlation

Consider a receive antenna array comprising N local ports. Following the theory presented in [8] it can be shown that in presence of spatially selective interference and additive white Gaussian noise, the signals available at the antenna ports are given by the vector,

$$\boldsymbol{y} = \mathbf{R}\boldsymbol{a}_{\mathrm{s}} + \mathbf{R}\boldsymbol{a}_{\mathrm{i}} + \boldsymbol{n}, \tag{1}$$

where $\mathbf{R} \in \mathbb{C}^{N\times M}$ is the matrix containing the antenna reception coefficients, $\boldsymbol{a}_{\mathrm{s}} \in \mathbb{C}^{M\times 1}$ is the vector containing the expansion coefficients of the incoming field in spherical vector waves modes corresponding to the main signal and, $\boldsymbol{a}_{\mathrm{i}} \in \mathbb{C}^{M\times 1}$ is the same for the interferer, where M is the maximum number of modes excited by the antenna. We further assume the channel is Rayleigh-fading with $\boldsymbol{a}_{\mathrm{s}} \sim \mathcal{C}\left(\mathbf{0}, \mathbf{C}_{a_{\mathrm{s}}}\right)$ and $\boldsymbol{a}_{\mathrm{i}} \sim \mathcal{C}\left(\mathbf{0}, \mathbf{C}_{a_{\mathrm{i}}}\right)$, i.e., the expansion coefficients are zero mean Gaussian variables with covariances $\mathbf{C}_{a_{\mathrm{s}}} = \mathbf{R}_{a_{\mathrm{s}}}$ and $\mathbf{C}_{a_{\mathrm{i}}} = \mathbf{R}_{a_{\mathrm{i}}}$, where $\mathbf{R}_a = E\left\{\boldsymbol{a}\boldsymbol{a}^\dagger\right\}$ is the correlation matrix. E denotes the expectation over the fading states and $(\cdot)^\dagger$ denotes Hermitian transpose. We also assume that the fields from the two signals are uncorrelated and that the noise covariance is $\mathbf{R}_n = \sigma_n^2\mathbf{I}$. Hence, the correlation matrix of $\boldsymbol{y}$ is

$$\mathbf{R}_y = \mathbf{R}\mathbf{R}_{\mathrm{ch}}\mathbf{R}^\dagger + \sigma_n^2\mathbf{I}, \tag{2}$$

where $\mathbf{R}_{\mathrm{ch}} = \mathbf{R}_{a_{\mathrm{s}}} + \mathbf{R}_{a_{\mathrm{i}}}$ is the correlation matrix of the channel.

The multi-poles are classified as TE ($\tau = 1$) or TM ($\tau = 2$). The azimuthal and radial dependencies are given by the m and l index, respectively. Whenever necessary, the multi-index $\iota \to \{\tau, m, l\}$ is introduced and identified with the number $\iota = 2(l^2 + l - 1 + m) + \tau$. Hence, the elements of the matrix, $\mathbf{R}_{\mathrm{ch}}$, can be further expressed as, $(\mathbf{R}_{\mathrm{ch}})_{\iota,\iota'} = (\mathbf{R}_{a_{\mathrm{s}}})_{\iota,\iota'} + (\mathbf{R}_{a_{\mathrm{i}}})_{\iota,\iota'}$ with

$$(\mathbf{R}_{a_{\mathrm{s}}})_{\iota,\iota'} = \frac{\varepsilon}{\varepsilon+1}\rho_{\iota,\iota'}(a_{\mathrm{s}}), \quad (\mathbf{R}_{a_{\mathrm{i}}})_{\iota,\iota'} = \frac{1}{\varepsilon+1}\rho_{\iota,\iota'}(a_{\mathrm{i}}), \tag{3}$$

where the correlation between the channel modes due to the polarization and statistical (spatial) distribution of the angle-of-arrival (AoA) of either the signal or the interferer is given by,

$$\rho_{\iota,\iota'}(a) = 4\pi(-\mathrm{i})^{l-l'-\tau+\tau'} \int \left(\frac{\chi_a p_{a,\theta}(\Omega) A^*_\iota(\Omega) A_{\iota',\theta}(\Omega) + p_{a,\phi}(\Omega) A^*_{\iota,\phi}(\Omega) A_{\iota',\phi}(\Omega)}{\chi_a + 1} \right) \mathrm{d}\Omega. \tag{4}$$

In (4), a stands for either a_{s} or a_{i}, $(\cdot)^*$ denotes the complex-conjugate, the elementary solid angle is given by $\mathrm{d}\Omega$, the signal-to-interference power ratio is $\varepsilon = P_{a_{\mathrm{s}}}/P_{a_{\mathrm{i}}}$ and the cross-polarization ratio (XPR) of the desired-signal channel, $\chi_{a_{\mathrm{s}}} = P_{a_{\mathrm{s}},\theta}/P_{a_{\mathrm{s}},\phi}$, and the interference channel, $\chi_{a_{\mathrm{i}}} = P_{a_{\mathrm{i}},\theta}/P_{a_{\mathrm{i}},\phi}$, respectively. The powers are normalized according to, $P_{a_{\mathrm{s}}} + P_{a_{\mathrm{i}}} = 1$ with $P_{a_{\mathrm{s}}} = P_{a_{\mathrm{s}},\theta} + P_{a_{\mathrm{s}},\phi}$ and $P_{a_i} = P_{a_{\mathrm{i}},\theta} + P_{a_{\mathrm{i}},\phi}$. The functions $p_{a_{\mathrm{s}},\theta}(\Omega)$, $p_{a_{\mathrm{i}},\theta}(\Omega)$, $p_{a_{\mathrm{s}},\phi}(\Omega)$, and $p_{a_{\mathrm{i}},\phi}(\Omega)$ denote the probability density functions (pdfs) of the AoAs of waves belonging to the desired-signal and the interference signal in both θ and ϕ polarizations, respectively. Thus, using the definition of the cross-correlation coefficient in [9], we obtain for branches with indices i and j,

$$\rho_{ij} = \frac{\left(\mathbf{R}\mathbf{R}_{\mathrm{ch}}\mathbf{R}^\dagger\right)_{i,j} + \sigma_n^2\delta_{ij}}{\sqrt{\left(\left(\mathbf{R}\mathbf{R}_{\mathrm{ch}}\mathbf{R}^\dagger\right)_{i,i} + \sigma_n^2\right)\left(\left(\mathbf{R}\mathbf{R}_{\mathrm{ch}}\mathbf{R}^\dagger\right)_{j,j} + \sigma_n^2\right)}}, \tag{5}$$

where $(\cdot)_{ij}$ denotes the matrix element with indices i and j.

As we can see from (3)-(5) the antenna branch cross-correlation is expressed through the radiation properties of the antennas and the polarization and directional properties of the propagation channel. The two expressions in the denominator of (5) are basically the power available at branch i and j, respectively. Hence, as expected, the correlation is a function of the link quality too.

3 Antennas and propagation scenario

Consider two half-wavelength dipoles in a slant-arrangement as shown in Fig. 1, where one dipole is tilted 45° from the z-axis towards the positive side of the y-axis, while the other is also tilted 45° but in the opposite direction. We denote them +45°- dipole and −45°- dipole, respectively. To study the correlation we rotate the antennas around the x-axis, towards the positive y-axis. The rotation angle is denoted by α. To illustrate the antenna channel interaction we consider generic models for the interference and the signal. We assume, that the probability density function (pdf) of the AoA is the same for both orthogonal polarizations and that they are independent in azimuth and elevation, $p_{\theta,\phi}(\theta,\phi) = p_\theta(\theta)\,p_\phi(\phi)$. We consider two pdfs for the AoA: 1) the 3D-uniform (isotropic) pdf, $p_{\theta,\phi}(\theta,\phi) = \sin\theta/4\pi$, and 2) the Laplacian pdf, $p_{\theta,\phi}(\theta,\phi) = A\exp\left(-\sqrt{2}\,|\theta-\mu_\theta|/\sigma_\theta - \sqrt{2}\,|\phi-\mu_\phi|/\sigma_\phi\right)\sin\theta$, with parameters σ_θ, μ_θ, σ_ϕ and μ_ϕ. We also assume that the XPR and the shapes of the pdfs of the main signal and the interferer signals are the same. The signal-to-interference power ratio equals $\varepsilon = \{0, 0.5, 1\}$. Based on these distributions four scenarios are considered with parameters specified in Table 1.

4 Simulation Results

As it is shown in Fig. 2, only the TM dipoles, i.e., modes with indices $l = 1$ and $\tau = 2$, are excited while the half-wavelength dipoles are rotated (the power that goes into the quadrupole modes, $l = 2$, is negligible). Indeed, rotation preserves the power in both the l and τ indices. We clearly see how the power in the TM

Table 1: Propagation and Interference Scenarios.

Sce.	Signal (or primary-cluster)	Interferer (or secondary-cluster)
A	Isotropic, $\chi_{\mathrm{dB}} = 0$	Isotropic, $\chi_{\mathrm{dB}} = 0$
B	Isotropic, $\chi_{\mathrm{dB}} = 0$	Laplacian, $\sigma_{a_i} = 0.1$, $\mu_{a_i} = \frac{\pi}{4}$, $\chi_{\mathrm{dB}} = 10$
C	Laplacian, $\sigma_{a_s} = 0.1$, $\mu_{a_s} = -\frac{\pi}{4}$, $\chi_{\mathrm{dB}} = 10$	Isotropic, $\chi_{\mathrm{dB}} = 0$
D	Laplacian, $\sigma_{a_s} = 0.1$, $\mu_{a_s} = -\frac{\pi}{4}$, $\chi_{\mathrm{dB}} = 10$	Laplacian, $\sigma_{a_i} = 0.1$, $\mu_{a_i} = \frac{\pi}{4}$, $\chi_{\mathrm{dB}} = 10$

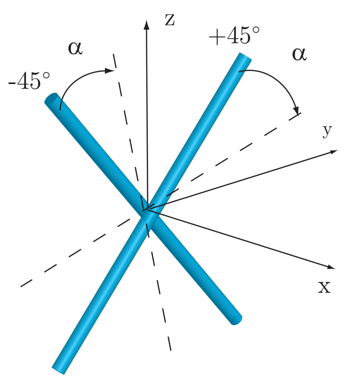

Figure 1: A $\pm 45°$ slanted polarization diversity arrangement. The rotation is towards the y-axis.

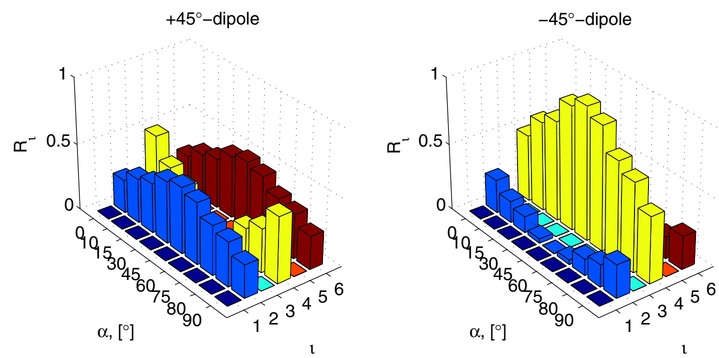

Figure 2: The squared absolute values of the reception (or transmission) coefficients, $|R_\iota|^2$, for the $+45°$- dipole and the $-45°$-dipole antennas as function of the rotation angle α and the ordered multi-pole multi-index $\iota = \{\tau, m, l\}$.

dipoles changes as the $+45°$ and the $-45°$ half-wavelength dipoles are tilted from $+45°$ to $+135°$ and from $-45°$ to $+45°$ with respect to the z-axis towards the y-axis, respectively. We immediately notice that at $\alpha = 45°$ the modes are orthogonal, hence the branch cross-correlation should be zero for any distribution of the field impinging at the antenna. At any other tilting the cross-correlation could also be zero, however, in that case it would depend on the properties of the impinging field. For example, Fig. 3 shows that the modes of the fields for the isotropic pdf of AoA are uncorrelated. On the other hand for AoA distributed according the Laplacian pdf with small angle-spread the field modes show much higher correlation, which implies higher correlation between the signals received at different antenna branches.

The impact of the pdf of the AoA as well as the XPR of the signal and the interferer on the correlation is shown in Fig. 4. Results are presented for propagation scenarios given in Table.1. The branch-correlation coefficient is presented as a function of the rotation angle, α and for three different values of the parameter ε, regulating the powers into the signal and the interferer. In scenario A the result is always uncorrelated branches, since modes of both the signal and the interferer are uncorrelated. Scenario B and C are similar due to the symmetry of the problem. In this case as the power of the Laplacian component, which is either the desired signal or the interfere for model B or model C, respectively, increases the branch correlation. However, the signals are uncorrelated for $\alpha = 45°$ as explained earlier. For model D, the branch correlation is independent of the parameter ε but depends upon the rotation angle α since the interaction between the antenna field modes and the modes of the field impinging at the antenna will change accordingly.

Acknowledgements

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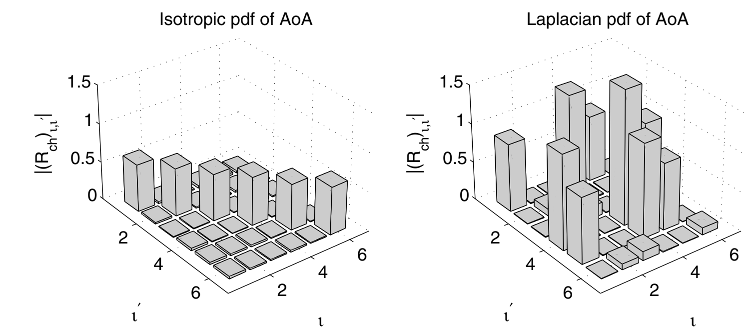

Figure 3: Channel covariance matrix.

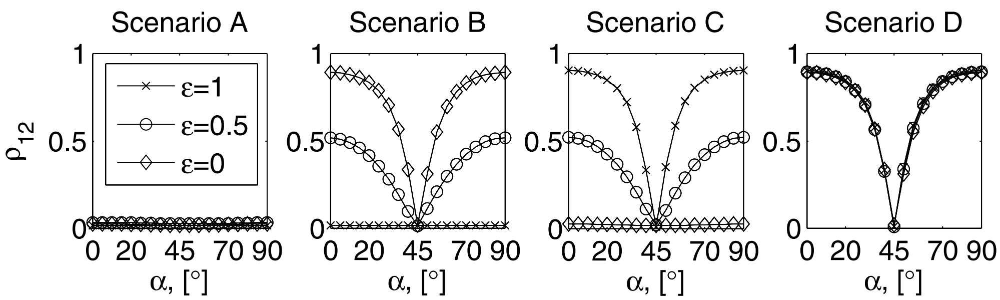

Figure 4: Branch correlation coefficient.

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