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Retrieval and Synthesis of Sources having a Circular Support and Generating Shaped Patterns

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Abstract

We propose a new approach to the retrieval or synthesis of continuous sources having a circular support and generating a given shaped power pattern. The framework can be applied for whatever (asymmetrical) distribution exhibited by the measured or desired power distribution. The method relies on overlooked tools from the aperture antenna theory as well as from known tools for array antenna synthesis.

1 Introduction

The design of array antennas and continuous aperture sources radiating shaped beams is a very long-standing subject in antenna synthesis [1]-[15]. A strictly related problem, which is also known as 'Phase retrieval' (PR), is that of retrieving a source from phaseless measurements of its near or far field. In the following, we mainly focus on the second of the two problems, i.e. PR, which arises in many areas of applied Sciences such as optics, electron microscopy, antennas, and crystallography (see [4] and the references cited therein).

The large number of design procedures which tackled such a problem proposed very many different ways in order to simplify it, including the exploitation of (just to cite a few): real field distributions [12], superimposed pencil beams [1], iterative FFT via virtual active element pattern expansion [11], a semidefinite relaxation of the unknowns [8], sequential Convex Programming (CP) [7], and power-pattern roots manipulation [14].

A related well-known approach is the Spectral Factorization (SF) technique, which was first developed in [2] and then extended to arrays being either isophoric [9] or fed by even excitations [3], to circularly-symmetric continuous sources [9], and to equispaced arrays having a high beam efficiency [10] or a rhombic equispaced layout [5]. As a matter of fact (see [13] for more details), the advantage provided by SF with respect to common approaches dealing with the *mask-constrained* power synthesis is the capability of: (i) ascertaining a-priori the actual feasibility of the problem, (ii) finding *all* the possible different array-excitation or aperture-field solutions, and (iii) casting the overall problem as a Linear Programming (LP) one plus a polynomial factorization.

Unfortunately, all the SF-based techniques published up to now (as well as most of the methods recalled above) exhibit a quite important limitation, i.e., they cannot be used every time the sought power pattern cannot be written in terms of a 1-D trigonometric polynomial. This is indeed the case in many important applications requiring, for instance, a source being neither one-dimensional nor circularly-symmetrical [13].

In the attempt of filling such a gap, this contribution deals with the PR of sources having a circular support from the square amplitude of its complex far field. The problem is cast in such a way to take advantage from all of the SF characteristics, while allowing the retrieving of generic planar continuous sources exhibiting no particular symmetries. This is possible by jointly exploiting the SF framework, the aperture antenna theory, and the electromagnetic field expansions reported and applied in [16],[17].

In the following, the proposed approach is presented in Section 2 and assessed in Section 3. Conclusions follow.

2 The Proposed Approach

The approach fully relies on the aperture antenna theory and, more in particular, on the expansions and formulas given in [17]. By virtue of those results, by denoting with (k',ϕ) and (ρ',ϕ') the radial and azimuth coordinates, respectively, in the spectral and spatial domains, the Fourier transform of a generic source $f(\rho',\phi')$ having a circular support of radius a can be expressed as (apart from unessential factors):

$$F(k',\phi) = = \frac{1}{2\pi}\int_0^{2\pi}\int_0^{a} f(\rho',\phi')e^{-jk'\rho'\cos(\phi'-\phi)}\rho' d\rho' d\phi' \qquad (1)$$

Moreover, the following relationships hold true [17]:

$$f(\rho',\phi') = \sum_{\ell=-L}^{L} f_\ell(\rho')e^{j\ell\phi'} \qquad (2)$$

$$F(k',\phi) = \sum_{\ell=-L}^{L} F_\ell(k')e^{j\ell\phi} \qquad (3)$$

where:

$$f_\ell(\rho') = \frac{1}{2\pi}\int_0^{2\pi} f(\rho',\phi')e^{-j\ell\phi'}d\phi' = \qquad (4)$$

$$= H_\ell^{-1}\{F_\ell(k')\} = \int_0^{\beta} F_\ell(k')J_\ell(k'\rho')k'\mathrm{d}k'$$

$$F_\ell(k') = \frac{1}{2\pi}\int_0^{2\pi} F(k',\phi)\, e^{-j\ell\phi} d\phi = \qquad (5)$$

$$= \int_0^a f_\ell(\rho') J_\ell(k'\rho')\rho' \mathrm{d}\rho' = H_\ell\{f_\ell(\rho')\}$$

where $\beta=2\pi/\lambda$ is the wavenumber (λ being the wavelength) and $k' = \sqrt{u^2+v^2} = \beta sin\theta$ ($u=\beta sin\theta cos\phi$, $v=\beta sin\theta sin\phi$), θ representing the aperture elevation angle with respect to the boresight. Finally, in (4) and (5) J_ℓ is the *ℓ-th* order Bessel function of first kind, and H_ℓ denotes the Hankel transform of order ℓ [16] of $f_\ell(\rho')$.

Note that in (2) and (3) the summation has been truncated by following the rules reported in [17],[18]. In particular, L is slightly larger than βa. It is also worth noting that the multipole expansions (2) and (3) allow expressing fields and sources having any azimuthal behavior, thus overcoming the limitations listed in Section 1. For example, $\ell=6$ and $\ell=0$ could be selected (see also [19]) in order to generate source having a hexagonally-symmetric behavior.

Notably, by virtue of the results in [2],[18], since the power pattern is a bandlimited function having twice the bandwidth of the corresponding complex field distribution, the square amplitude of (3) can be written as a linear combination of $4L+1$ complex coefficients, say $D_{-2L}(k'), \ldots, D_0(k'), \ldots, D_{2L}(k')$, i.e.:

$$P(k',\phi) = \sum_{p=-2L}^{2L} D_p(k')\, e^{jp\phi} \qquad (6)$$

By straightforward derivations, expression (6) can also be analyzed as the restriction to the unit circle of a polynomial in the complex variable $z = e^{ju}$ [2]. As $P(k',\phi)$ must be a real and non-negative function, one also has:

$$D_p(k') = D^*_{-p}(k') \quad p = 1,\ldots,2L \quad \forall k' \qquad (7)$$

$$\sum_{p=-2L}^{2L} D_p(k')\, e^{jp\phi} \geq 0 \qquad \forall k',\phi \qquad (8)$$

* meaning complex conjugation.

In case of PR problems, the power pattern $P(k',\phi)$ is derived from measurements. Under such assumptions, the overall procedure consists of the three following steps:

1. Determine the complex coefficients $D_p(k')$ in such a way that the representation (8) matches the given power pattern $P(k',\phi)$ and the properties (7) and (8) are satisfied;
2. once the $D_p(k')$ are obtained, determine the $F_\ell(k')$ functions (see the following);
3. apply (2) and (4) to the identified $F_\ell(k')$ functions and determine the source distribution $f(\rho',\phi')$.

Notably, step 1 amounts in solving a *LP problem*, with the inherent advantages in terms of computational burden and optimality of results. Conversely, step 2 is the most difficult part of the problem. In order to solve it, at least two different strategies may be pursued, as detailed in the following.

A first possibility relies on the fact that, since it is $P(k',\phi) = |F(k',\phi)|^2$, $F_\ell(k')$ can be retrieved for each fixed value of $\tilde{k}'$ by factorizing (6) into two complex-conjugate factors, i.e.,

$$P(\tilde{k}',\phi) = F(\tilde{k}',\phi)F^*(\tilde{k}',\phi) \qquad (9)$$

in exactly the same way as in [2]. Notably, the solution to each of these problems is not unique, so that, by reasoning as in [20], one has to make some selection in order to provide a congruence amongst the solution selected for each value of k'.

A second possibility relies on the fact that, since it is $P(k',\phi) = |F(k',\phi)|^2$, the coefficients $D_p(k')$ represent the autocorrelation of $F_\ell(k')$ and hence the following relationships hold true $\forall k'$:

$$\begin{cases} D_{2L}(k') = F_L(k')F^*_{-L}(k') \\ D_{2L-1}(k') = F_L(k')F^*_{-L+1}(k') + F_{L-1}(k')F^*_{-L}(k') \\ D_{2L-2}(k') = F_L(k')F^*_{-L+2}(k') + F_{L-1}(k')F^*_{-L+1}(k') \\ F_{L-2}(k')F^*_{-L}(k') \\ \ldots \\ D_{-2L}(k') = F_{-L}(k')F^*_L(k') \end{cases} \qquad (10)$$

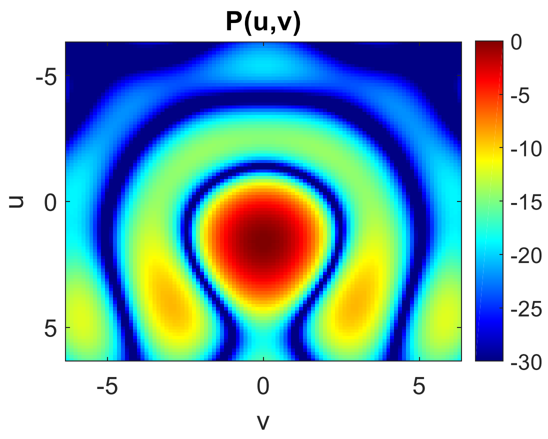
Then, one has to solve the above set of equations in the unknowns $F_\ell(k')$. In so doing, one can take advantage of the expected properties of $F_\ell(k')$ and $D_p(k')$ (i.e., each of these functions has a zero of order ℓ in the origin [17]). Moreover, a relatively simple solution approach is viable in case $F_\ell(k') = F_{-\ell}(k')$. In fact, under such an assumption, the first equation can be solved by performing a 1-D spectral factorization (along the guidelines in [2]) and the resulting solutions can be substituted into the subsequent equations.

3 Numerical Results

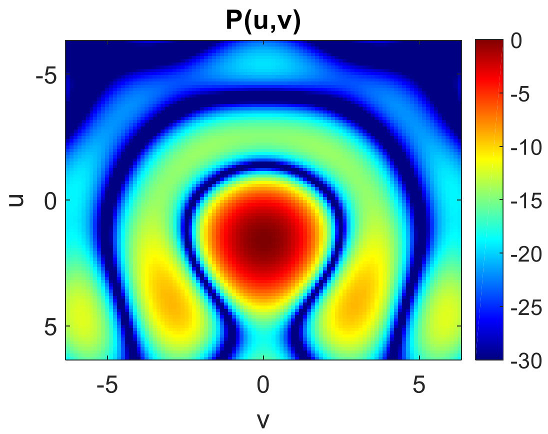
In order to present a preliminary assessment of the proposed approach, we performed the PR of field being even with respect to the v axis and radiated by a circular aperture of radius a=1.25λ.

The reference and retrieved power patterns are shown in Fig. 1. The first equation in (10) has been optimally solved by representing $D_{2L}(k')$ as a trigonometric polynomial. Then, the latter has been factorized in such a way to determine $F_L(k')$. Finally, the subsequent nonlinear equations in (10) have been simultaneously solved through the *fsolve* function of Matlab in order to determine the unknown functions $F_\ell(k')$ ($\ell = L-1,\ldots,0$). As it can been seen in Fig. 2, the reconstructed functions $D_\mathrm{p}(k')$ result essentially equal to the ones associated to the reference pattern.

The reported result, as well as the others which will be shown during the conference, confirm the interest of the proposed approach. The runtime for the overall procedure turned out to be less than 2 minutes (by using a computer equipped with a 2.21 GHz CPU and 16GB of RAM memory).

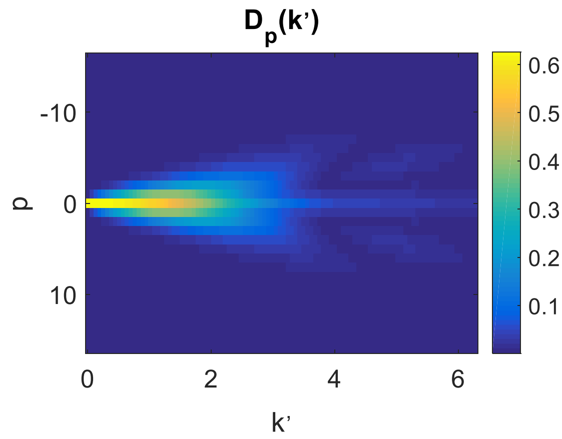


(a)

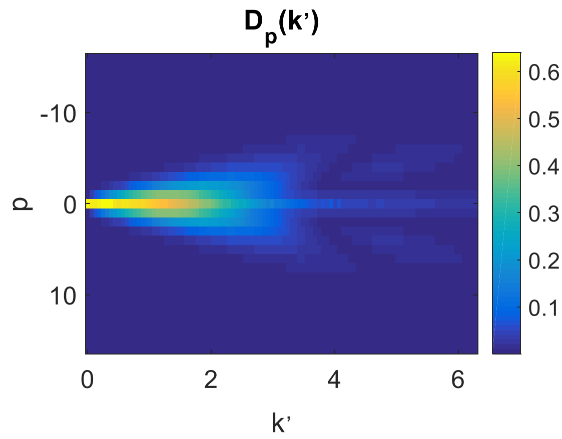


(b)

Figure 1. Reference (a) and retrieved (b) power patterns.



(a)



(b)

Figure 2. Real part of the reference (a) and retrieved (b) coefficients $D_p(k')$, denoting that the PR procedure has been successfull.

4 Conclusions

A new approach, relying on the aperture antenna theory, the spectral factorization, and a multipole expansion of the far-field and source distributions has been devised for the problems of retrieving or synthesizing sources having a circular support generating given (power) shaped beams.

5 References

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