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Quantum Computational Electromagnetics

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Abstract

We investigate quantum information preserving computational electromagnetics. To do so, we propose two different methods which are (1) numerical canonical quantization and (2) quantum finite-difference time-domain scheme. The proposed algorithms find numerical solutions of quantum Maxwell's equations involved in arbitrary inhomogeneous dielectric media based on the macroscopic theory on quantum electrodynamics; hence, they are useful to study the quantum nature of electromagnetic fields in arbitrary passive and lossless quantum optical instruments. We provide a numerical example associated with two photon interference occurring in a quantum beam splitter, which is the underlying principle of recent quantum technologies.

1 Introduction

The recent advent in quantum computing spells the beginning of an exciting era for quantum technologies. Mathematical modeling of physical phenomena and their numerical simulations have transformed classical electromagnetics technologies. But such knowledge base is still in its infancy for quantum Maxwell's equations and quantum technologies. Historically, after the discovery of the quantum nature in atoms, the development of quantum optics has been triggered by Dirac and is still under active investigation until now. It mainly deals with the physics of single photon, which has energy quanta. It carries the quantum information while riding on electromagnetic (EM) fields. As a result, EM fields are to be also quantized in the sense of the quantum mechanics. Their quantum elevation is typically done via the (monochromatic) mode decomposition picture. Called canonical quantization, it uses a complete basis set to spatially separate total fields on Fourier space. As such that classical field amplitudes are simply replaced by operators acting on wavefunctions of photons. One of most important examples showing the quantum nature of EM fields is a quantum beam splitter. It is obvious that the deterministic behavior is present in a classical 50/50 beam splitter that evenly divides input light beam into two reflected and transmitted beams simultaneously measurable at output ports. Every single photon passing through a quantum beam splitter, however, randomly arrives at one of two outputs, as illustrated in Fig. 1. This bizarreness has been manifested in experiments by Grangier and Hong-Ou-Mandel.

In this conference paper, we explore using computational electromagnetic techniques (CEM) to capture and characterize the quantum nature of quantum beam splitters. It should be mentioned that recent efforts [1] sought to build bridges between the engineering and physics community to foster cross pollination. This work is along the same line of [1] but with more focus. Specifically, we propose (1) numerical canonical quantization and (2) quantum finite-difference time-domain (Q-FDTD) scheme. The former method is the extended version of standard canonical quantization where now modes are formed numerically instead of using Fourier modes. In other words, it quantizes EM fields in the presence of arbitrary inhomogeneous dielectric media. Therein, instead of finding *ad-hoc* normal modes in closed form, we numerically solve the corresponding Helmholtz wave equation on a given mesh by using either finite-difference/-element methods with Bloch-periodic boundary conditions. For the latter, on the other hand, we look at the quantum nature of EM fields from a different angle, i.e. in the coordinate picture, to develop a time-domain quantum Maxwell solver. In the Heisenberg picture, quantum Maxwell dynamical operators are expanded by a set of ladder operators defined in the coordinate space while non-orthogonal basis corresponding to a new type of propagators which are different from the classical Green's function. In particular, the proposed time-domain quantum Maxwell solver is useful to study the local propagation of quantum information such as virtual photons or near single photon sources. Using both methods, we provide numerical simulation results associated with two photon interference occurring in a quantum beam splitter.

2 Numerical canonical quantization

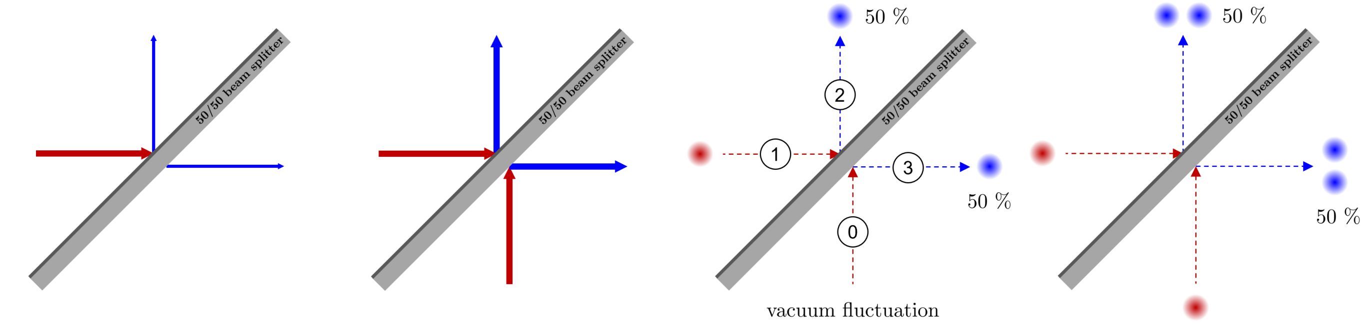
For a 1-D periodic box including arbitrary dielectric objects, singly-linearly-polarized vector potential operators can be represented by using a countably-infinite number of normal modes for the system [2, 3] as

$$\hat{A}(x,t) = \hat{A}^{(+)}(x,t) + \hat{A}^{(-)}(x,t), \tag{1}$$

$$\hat{A}^{(+)}(x,t) = \sum_{p} \sqrt{\frac{\hbar}{2\omega_p}} \phi^{(p)}(x) e^{-i\omega_p t} \hat{a}_{k_p}, \tag{2}$$

$$\hat{A}^{(-)}(x,t) = \sum_{p} \sqrt{\frac{\hbar}{2\omega_p}} \phi^{(p)*}(x) e^{i\omega_p t} \hat{a}^{\dagger}_{k_p}, \tag{3}$$

where dummy integer p denotes normal mode index, $\phi^{(p)}(x)$ is p-th normal mode, ω_p and k_p are eigen-



(a) Single input of classical light beam **(b)** Both inputs of classical light beams **(c)** Single-photon incidence **(d)** Two-photon incidence

Figure 1. Schematics for classical and quantum beam splitters for various input scenarios.

frequency and -wavenumber, and $\hat{a}_{k_p}$ and $\hat{a}^{\dagger}_{k_p}$ are annihilation and creation operators in the mode space, respectively. The normal modes are solutions of the Helmholtz wave equation with the use of the generalized transverse gauge which is given by

$$\frac{d^2}{dx^2}\phi^{(p)}(x) + \varepsilon(x)\mu_0\omega_p^2\phi^{(p)}(x) = 0 \tag{4}$$

where the dielectric inhomogeneity is incorporated into $\varepsilon(x)$. The ladder operator is to be acting on a quantum state which can be made by the linear superposition of multimode Fock states.

One can numerically solve (4) by using either finite-element or finite-difference methods for a given mesh. The discrete counterpart of (4) can be written by

$$\bar{\mathbf{S}}\cdot\bar{\Phi} + \bar{\mathbf{M}}\cdot\bar{\Phi}\cdot\bar{\lambda} = 0 \tag{5}$$

where $\bar{\mathbf{S}}$ and $\bar{\mathbf{M}}$ are stiffness and mass matrices which are Hermitian, $\bar{\Phi}$ and $\bar{\lambda}$ are matrices including a set of eigenvectors and eigenvalues whose elements are given by

$$\left[\bar{\Phi}\right]_{i,p} = \phi^{(p)}(x_i), \tag{6}$$

$$\left[\bar{\lambda}\right]_{p,p} = \omega_p^2. \tag{7}$$

Note that the total number of eigenvectors and eigenvalues is determined by the total number of grid points. With the use of Bloch-periodic boundary condition, numerical normcal modes can be complex-valued, i.e., in the form of traveling waves. Consequently, the corresponding vector potential operators can be expanded by a countably-finite set of numerical normal modes and written in the matrix representation as

$$\bar{\hat{\mathbf{A}}}(t) = \bar{\Phi}\cdot\bar{\mathbf{D}}(t)\cdot\hat{\mathbf{a}} + \hat{\mathbf{a}}^{\dagger}\cdot\bar{\mathbf{D}}^{\dagger}(t)\cdot\bar{\Phi}^{\dagger} \tag{8}$$

where † denotes Hermitian conugate,

$$\left[\bar{\mathbf{A}}(t)\right]_i = \hat{A}(x_i,t), \tag{9}$$

$$\left[\hat{\mathbf{a}}\right]_p = \hat{a}_{k_p}. \tag{10}$$

We call this *numerical canonical quantization* which allows one to obtain Maxwell dynamical operators for arbitrary dielectric inhomogeneous media. More details can be found in [3].

3 Initial quantum state

An operator itself can have physical meaning only if it operates on a vector. Here, we briefly discuss how to make a quantum state for a single photon riding on wavepacket. Due to the bosonic property of photons, one can construct an initial quantum state for a single photon riding on wavepacket through multimode Fock states as

$$\left|\Psi^{(1)}\right\rangle = \sum_p \tilde{g}_p\hat{a}^{\dagger}_{k_p}\left|0\right\rangle \tag{11}$$

where $\tilde{g}_p$ is a probability amplitude incorporating the spectral information of a (gaussian) wavepacket and $|0\rangle$ denotes a vacuum state in the Dirac's notation. Furthermore, if two photons are independent (not entangled), their composite initial quantum state can be given by the tensor product of each single photon's quantum state, viz.,

$$\left|\Psi^{(2)}\right\rangle = \left[\sum_{p'}\tilde{g}^{(B)}_{p'}\hat{a}^{\dagger}_{k_{p'}}\right]\left[\sum_{p}\tilde{g}^{(A)}_{p}\hat{a}^{\dagger}_{k_{p}}\right]\left|0\right\rangle. \tag{12}$$

4 Quantum FDTD scheme

Since the numerical normal modes satisfy the orthonormal property, which can be written in the matrix representation as

$$\bar{\Phi}^{\dagger}\cdot\bar{\mathbf{M}}\cdot\bar{\Phi} = \bar{\mathbf{I}} \tag{13}$$

where $\bar{\mathbf{I}}$ is identity matrix, one can associate ladder operators in the mode space with those in the coordinate space via the unitary transformation

$$\hat{\mathbf{b}} = \bar{\mathbf{M}}\cdot\bar{\Phi}\cdot\hat{\mathbf{a}} \tag{14}$$

where $\left[\hat{\mathbf{b}}\right]_v = \hat{b}_{x_v}$. Substituting (14) into (8), one can represent the vector potential operator with respect to coordinate-ladder operators

$$\bar{\hat{A}}(t) = \bar{\mathbf{G}}(t)\cdot\hat{\mathbf{b}} + \hat{\mathbf{b}}^{\dagger}\cdot\bar{\mathbf{G}}(t)^{\dagger} \tag{15}$$

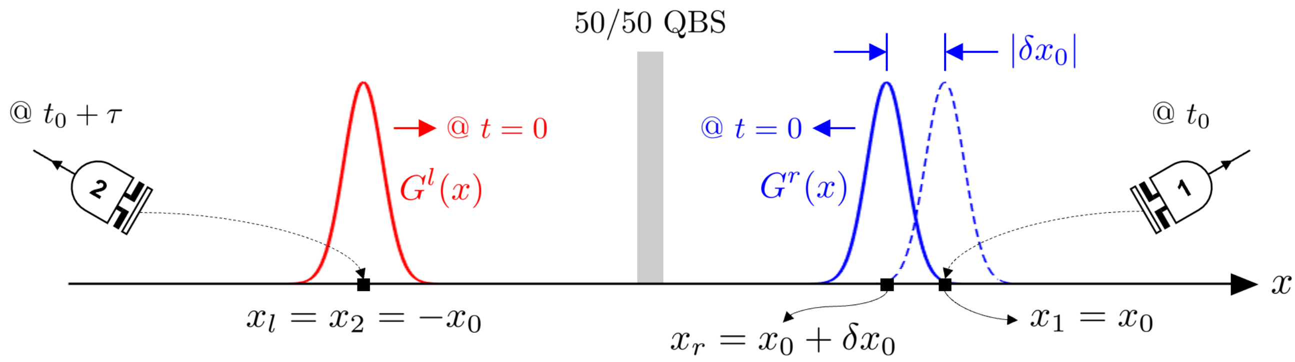


Figure 2. Schematic of numerical experiments for two-photon incidence to observe Hong-Ou-Mandel effect.

where

$$\left[\bar{\mathbf{G}}(t)\right]_{i,\nu}=G_\nu(x_i,t), \tag{16}$$

$$G_\nu(x_i,t)=\sum_p\sqrt{\frac{\hbar}{2\omega_p}}\phi^{(p)}(x_i)\,\phi^{(p)^*}(x_\nu)\,e^{-i\omega_p t}. \tag{17}$$

The function $G_\nu(x_i,t)$ is interpreted as a new propagator carrying fields through space and time, which are initiated from a certain grid point x_ν. This is different from the classical Green's function due to the presence of the factor $\sqrt{\hbar/(2\omega_p)}$. Since coordinate-ladder operators are not a function of space and time, vector potential operators is time-evolving through the new propagator. Hence, by numerically time-evolving the new propagator, one deduces the time evolution of vector potential operators. Since the new propagator is a solution of wave equation, one can numerically time-evolve ν-th propagator through the following Q-FDTD scheme as

$$\frac{[G_\nu]^n_{i+1}-2[G_\nu]^n_i+[G_\nu]^n_{i-1}}{\Delta x^2} -\varepsilon_i\mu_0\frac{[G_\nu]^{n+1}_i-2[G_\nu]^n_i+[G_\nu]^{n-1}_i}{\Delta t^2}=0 \tag{18}$$

for $i=1,2,\cdots,N_g$ and $\nu=1,2,\cdots,N_g$ where N_g is the total number of grid points and $[G_\nu]^n_i\equiv G_\nu(x_i,t_n)$. Note that there is no coupling with respect to different ν.

It should be mentioned that since coordinate ladder operators retain the same properties of mode ladder operators and commutator relation, one can easily find initial quantum states for coordinate ladder operators by applying the unitary transformation to (11) or (12).

5 Numerical example

Two-photon interference in a quantum beam splitter, called Hong-Ou-Mandel (HOM) effect, is the underlying principle of quantum technologies, yielding quantum entangled states. The more identical the pair of two incident photons are, the smaller the probability of simultaneous detection becomes at output ports of quantum beam splitters. Hence, it implies the degree of indistinguishability of incident photons in terms of various properties such as frequency, bandwidth, time delay, and so forth. Indistinguishability of two photons is often measured through the degree of intensity coherence, called second order correlation, denoted as $g^{(2)}$. It was firstly reported in [4] for the development of advanced stellar interferometers with classical light beam inputs. The quantum version of the second order correlation function can be deduced from the physical mechanism of the simultaneous detection of two photons [2]. In other words, the action of an annihilation operator to a quantum state resembles absorption of a single photon at a specific location and certain time instant, viz., the photoelectric effect. The detailed derivation for the quantum second order correlation function can be found in [5][Chap. 12]. Hence, $g^{(2)}$ is widely used in many quantum-associated experiments to test the indistinguishability of two photons.

To test indistinguishability of two photons, we place two photodetectors at x_1 and x_2, as illustrated in Fig. 2. Each photodetector is supposed to detect a photon at a certain time instant $t_0=t_1$ and $t_1+\tau$, respectively. Note that τ is equal to $\delta x_0/c$ where c is the speed of light. The time instant $g^{(2)}$ can be written as

$$g^{(2)}(x_1,t;x_2,t+\tau)=\frac{A}{B_1C_2} \tag{19}$$

where

$$A=\left\langle\Psi^{(2)}\middle|\hat{A}^{(-)}(x_1,t_1)\hat{A}^{(-)}(x_2,t_1+\tau)\times\hat{A}^{(+)}(x_2,t_1+\tau)\hat{A}^{(+)}(x_1,t_1)\middle|\Psi^{(2)}\right\rangle, \tag{20}$$

$$B_1=\left\langle\Psi^{(2)}\middle|\hat{A}^{(-)}(x_2,t_1)\hat{A}^{(+)}(x_2,t_1)\middle|\Psi^{(2)}\right\rangle, \tag{21}$$

$$B_2=\left\langle\Psi^{(2)}\middle|\hat{A}^{(-)}(x_2,t_1+\tau)\hat{A}^{(+)}(x_2,t_1+\tau)\middle|\Psi^{(2)}\right\rangle. \tag{22}$$

We set $x_1=x_r=x_0$ and $x_2=x_l=-x_0$. Note that A, B_1, and B_2 can be explicitly evaluated by using the rule of the action of ladder operators to number states and commutator relation.

Fig. 3 illustrates $g^{(2)}(\tau)$ versus τ evaluated by using numerical canonical quantization for different grids and numerical methods. Note that $N^{(0)}$ denotes the total number of grid

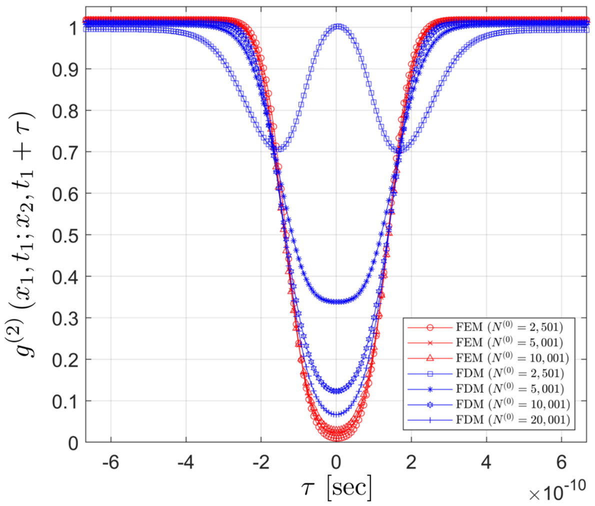


Figure 3. The Hong-Ou-Mandel effect obtained by numerical canonical quantization for different meshes and numerical methods.

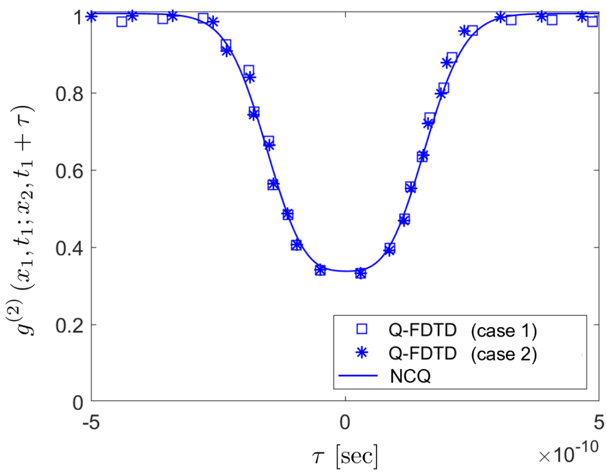


Figure 4. Comparison of the HOM effects obtained by numerical canonical quantization (NCQ) and Q-FDTD scheme when $N^{(0)} = 5001$.

points. In both methods, as the mesh becomes more refined, the dip around $\tau = 0$ approaches to zero, called HOM dip, with different convergence rates. This is the clear evidence of two photons emerging through either output ports while being bunched since the pair of photons are perfectly identical; hence, they are indistinguishable. It should be mentioned that the HOM dip is usually less than one half, as evidenced in experimental results. This is entirely a quantum effect and cannot be explained by (semi-)classical theory that produces the dip always greater than one half depending on the type of incident fields such as coherent pulses or chaotic lights [6, 7].

Furthermore, for $N^{(0)} = 5001$, we ran the Q-FDTD scheme for the same problem. Results are compared in Fig. 4 with numerical canonical quantization (NCQ) based on finite-difference method. Note that case 1 and 2 are slightly different in terms of constructing initial quantum states. It can be seen that there are great agreement among all cases.

6 Acknowledgements

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