Time evolution around exceptional points

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Exceptional points (EPs) are a type of spectral degeneracy specific to non-hermitian models [1], which can be used to describe active systems, such as paraxial waveguides or coupled cavities with gain and loss. The peculiar physical properties of EPs have been theorised to possibly give rise to superior sensors [2] and polarisers [3]. Given the experimental difficulty of operating at an EP, a lot of attention has been devoted to potential workarounds such as dynamical EP encircling [3].

Here, we show that:

- EPs are not drastically different from points in their vicinity.
- Time-modulated non-hermitian systems exhibit extremely rich physics whose complexity is not captured by an EP-encircling rule.

We begin with time-independent non-Hermitian systems, by explaining how time evolution works in the absence of a full set of eigenmodes (i.e. at the EP). Then, we show that this behaviour is simply a limit of general non-hermitian time evolution. We continue with a classification of the evolution of polarisation under two-level non-hermitian Hamiltonians based on the Möbius group and show that even in generic cases, any initial condition converges to a fixed point. We conclude that most non-Hermitian systems can act as polarisers.

Building on this understanding of static non-hermitian Hamiltonians, we turn to time-modulated ones and derive exact solutions for a large class of modulation curves, which comprises any piecewise-defined curve in the complex plane with circular, linear and parabolic elements. This extends the toolbox of analytical solutions, allowing for closer comparison with experimentally feasible time modulations. When the modulation is periodic in time, the results can be interpreted within the Floquet picture. Importantly, the non-periodic envelope of the evolution operator can behave very differently from the evolution associated to neighbouring static Hamiltonians. For instance, Floquet EPs can arise for time modulations that are neither close to nor encircle the static EP. We can therefore control and generate EPs simply by designing the modulation scheme.

Having thus shown that the behaviour of time-modulated non-hermitian systems is highly dependent on the precise shape of the modulation curve, we start exploring the elaborate interplay between non-hermiticity and parametric resonance through three families of modulation trajectories, which respectively correspond to a classical parametric resonance, a case dominated by non-hermiticity and an intermediate situation. These investigations reveal intriguing phenomena, such as the merging of stability peaks in Strutt diagrams or the non-trivial distribution of Floquet EPs in parameter space.

References

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