Multiple scattering by two PEC sphere

S. Batool*(1)(2), M. Nisar(1)(2), F. Mangini(3), F. Frezza(1)(2) E. Fazio(1)(2)
(1) Department of Information Engineering, Electronics and Telecommunications,
"La Sapienza" University of Rome, Via Eudossiana 18, 00184 Rome, Italy
(2) Department of Fundamental and Applied Sciences for Engineering,
"La Sapienza" University of Rome, Via A. Scarpa 16, 00161 Roma, Italy
(3) Department of Information Engineering, University of Brescia,
Via Branze 59, 25123 Brescia, Italy

Abstract

An analysis of multiple scattering by two Perfect Electric Conducting (PEC) sphere using translation Addition Theorem (AT) for spherical vector wave equation is presented. Specifically, Cruzan formalism is used to represent the AT for spherical harmonics, which introduced translation coefficients for transformation of spherical harmonic from one coordinate to another. These coefficients allow a very efficient solution for the calculation of multiple scattering electric field using two PEC sphere in a near zone region. The numerical simulations are included the validity of the multiple scattering behavior.

1 Introduction

Multiple scattering of electromagnetic waves is very essential for researcher in several fields e.g. optical diagnostic methods [1, 2, 3], solid state physics [4], radar and remote sensing etc... [5]. Generally, the scattering by a finite multiple bodies is complicated. Several analysis has been conducted into the multiple scattering by PEC sphere using translation addition theorem for spherical harmonics. However, the modern computational approaches have been enabled us to investigate deeply the random and rough surfaces, as a consequence of numerical simulation allow us to determine the scattering techniques [6, 7, 8, 9].

Liang et. al., [10] was the first scientist, who described the solution of Maxwell equation for the connected spheres. He purposed the superposition technique for the analysis of internal or external field solution by an individual sphere in terms of vector spherical harmonics. Most recently Mackowski described the further improvement in the superposition solution method for multiple sphere scattering problems [11].

Xu expresses the multiple scattering by spheres. Although, Mie-theory is very important for the solution of scattering electric field in terms of spherical harmonic wave function [12]. Since in 1954, Friedman and Russek described the calculation of addition theorem for spherical scalar wave function [13]. Further, Cruzen and Stein developed addition theorem for spherical vector wave functions. It becomes feasible for the appropriate solution of multiple scattering by spheres [14, 15].

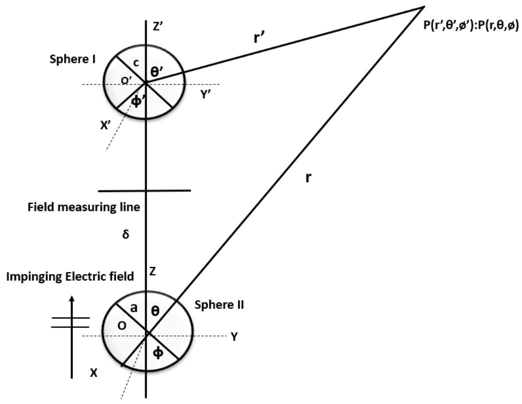

Figure 1. Geometry of the two PEC sphere exercising translation along the $z-axis$.

In our manuscript, we study the multiple scattering analysis using two PEC sphere. We have selected the Cruzen formalism for the translation addition coefficients. We have derived scattering electric field using AT for spherical harmonics. We have computed mathematical formation using numerical simulation approach.

2 Formation

Let us consider two PEC spheres having radii a and c with respect to the center at the origins O and O' of two different coordinate systems. So, spherical coordinate system (r, θ, ϕ) refer to the origin O and (r', θ', ϕ') refer to the origin O', respectively. The distance between the center of spheres along z-axis is δ. A linearly polarized plane wave impinging along z-axis. Geometry of the two PEC sphere performing translation along the $z-axis$ using translation AT for spherical vector wave function shown in Fig. 1.

3 Expansion of incident plane wave

The incident plane wave is expanded into the multipole fields series surrounded by the origin O and O' of the spherical coordinate systems (r,θ,ϕ) and (r',θ',ϕ'), Since the radial vector $r' = \exp(-ik_0 r)\exp(-ik_0\cos\alpha)$ and $r = r' + b$. Incident plane wave using multipole coefficients expansion using spherical coordinates may be written as

$$\mathbf{E}_i(\mathbf{r}',\theta',\phi') = \sum_{\nu=1}^{+\infty}\sum_{\mu=-\nu}^{\nu} \left[a^*_{\mu\nu}\mathbf{M}^{(1)}_{\mu\nu}(\mathbf{r}',\theta',\phi') + b^*_{\mu\nu}\mathbf{N}^{(1)}_{\mu\nu}(\mathbf{r}',\theta',\phi')\right] \tag{1}$$

$$a^*_{\mu\nu} = \exp(-ik_0\cos\alpha)a_{\mu\nu} \tag{2}$$

$$b^*_{\mu\nu} = \exp(-ik_0\cos\alpha)b_{\mu\nu} \tag{3}$$

4 Scattered field by two PEC sphere

Scattered field by two PEC spheres may be represented as $\mathbf{E}^I_s$ and $\mathbf{E}^{II}_s$, respectively. In this problem, the total electric fields are described by the incident and scattered electric fields. So the following expression of the total electric field may be written as:

$$\mathbf{E}^{total} = \mathbf{E}_i + \mathbf{E}^I_s + \mathbf{E}^{II}_s \tag{4}$$

where

$$\mathbf{E}^I_s(r,\theta,\phi) = \sum_{\nu=1}^{+\infty}\sum_{\mu=-\nu}^{\nu} \left[e_{\mu\nu}\mathbf{M}^{(3)}_{\mu\nu}(\mathbf{r},\theta,\phi) + f_{\mu\nu}\mathbf{N}^{(3)}_{\mu\nu}(\mathbf{r},\theta,\phi)\right] \tag{5}$$

$$\mathbf{E}^{II}_s(r',\theta',\phi') = \sum_{\nu=1}^{+\infty}\sum_{\mu=-\nu}^{\nu} \left[g_{\mu\nu}\mathbf{M}^{(3)}_{\mu\nu}(\mathbf{r}',\theta',\phi') + h_{\mu\nu}\mathbf{N}^{(3)}_{\mu\nu}(\mathbf{r}',\theta',\phi')\right] \tag{6}$$

5 AT for spherical vector wave equation

From AT, the translation of the vector spherical harmonics depends on the relative separation in a spherical coordinate system with respect to the different origin O and O'. To express the translation of l_{th} coordinates system (r,θ,ϕ) to j_{th} (r',θ',ϕ') coordinate system, we have:

$$\mathbf{M}^{(3)}_{\mu\nu}(l) = \sum_{n=1}^{\infty}\sum_{m=-n}^{n} A^{\mu\nu}_{mn}(l,j)\mathbf{M}^{(1)}_{mn}(j) + B^{\mu\nu}_{mn}(l,j)\mathbf{N}^{(1)}_{mn}(j) \tag{7}$$

$$\mathbf{N}^{(3)}_{\mu\nu}(l) = \sum_{n=1}^{\infty}\sum_{m=-n}^{n} B^{\mu\nu}_{mn}(l,j)\mathbf{M}^{(1)}_{mn}(j) + A^{\mu\nu}_{mn}(l,j)\mathbf{N}^{(1)}_{mn}(j) \tag{8}$$

Translation of a j_{th} coordinate system (r',θ',ϕ') to l_{th} coordinates system (r,θ,ϕ) at translation distance δ may be described as:

$$\mathbf{M}^{(3)}_{\mu\nu}(j) = \sum_{n=1}^{\infty}\sum_{m=-n}^{n} C^{\mu\nu}_{mn}(j,l)\mathbf{M}^{(1)}_{mn}(l) + D^{\mu\nu}_{mn}(j,l)\mathbf{N}^{(1)}_{mn}(l) \tag{9}$$

$$\mathbf{N}^{(3)}_{\mu\nu}(j) = \sum_{n=1}^{\infty}\sum_{m=-n}^{n} D^{\mu\nu}_{mn}(j,l)\mathbf{M}^{(1)}_{mn}(l) + C^{\mu\nu}_{mn}(j,l)\mathbf{N}^{(1)}_{mn}(l) \tag{10}$$

6 Calculation of vector translation coefficients for two PEC spheres

Suppose that an electromagnetic field in terms of translation is represented by infinite sum with respect to a coordinates system toward a different reference. So, Cruzan's mathematical expressions for $A^{l,j}_{mn\mu\nu}$ and $B^{l,j}_{mn\mu\nu}$ can be written as:

$$A^{l,j}_{mn\mu\nu} = (-1)^{-m}\frac{(2\nu+1)(n+m)!(\nu-\mu)!}{2n(n+1)(n-m)!(\nu+\mu)!}\exp[i(\mu-m)\phi_{lj}] \times \sum_{q=0}^{q_{max}} i^p[n(n+1)+\nu(\nu+1)-p(p+1)]a_q \times h^{(1)}_p(kd_{lj})P^{\mu-m}_p(\cos\theta_{lj}) \qquad r' < r \tag{11}$$

Similarly

$$A^{l,j}_{mn\mu\nu} = (-1)^{-m}\zeta(m,n,\mu,\nu)\sum_{q=0}^{q_{max}} h^{(1)}_p(kr) \tag{12}$$

$$C^{j,l}_{mn\mu\nu} = (-1)^{-m}\zeta^*(m,n,\mu,\nu)\sum_{q=0}^{q_{max}} h^{(1)}_p(kr) \tag{13}$$

where

$$\zeta(m,n,\mu,\nu) = \frac{(2\nu+1)(n+m)!(\nu-\mu)!}{2n(n+1)(n-m)!(\nu+\mu)!}\exp[i(\mu-m)\phi_{lj}] \times \sum_{q=0}^{q_{max}} i^p[n(n+1)+\nu(\nu+1)-p(p+1)]a_q \quad P^{\mu-m}_p(\cos\theta_{lj})$$

and:

$$B^{l,j}_{mn\mu\nu} = (-1)^{-m+1}\frac{(2\nu+1)(n+m)!(\nu-\mu)!}{2n(n+1)(n-m)!(\nu+\mu)!}\exp[i(\mu-m)\phi_{lj}] \times \sum_{q=1}^{Q_{max}} i^{p+1}\{[(p+1)^2-(n-\nu)^2][(n+\nu+1)^2-(p+1)^2]\}^{\frac{1}{2}} \times b(-m,n,\mu,\nu,p+1,p)h^{(1)}_{p+1}(kr) \times P^{\mu-m}_{p+1}(\cos\theta_{lj}) \qquad r' < r \tag{14}$$

Similarly

$$B^{l,j}_{mn\mu\nu} = (-1)^{-m+1}\xi(m,n,\mu,\nu)\sum_{q=0}^{q_{max}} h^{(1)}_{p+1}(kr) \tag{15}$$

$$D^{l,j}_{mn\mu\nu} = (-1)^{-m+1}\xi^*(m,n,\mu,\nu)\sum_{q=0}^{q_{max}} h^{(1)}_{p+1}(kr) \tag{16}$$

where

$$\xi(m,n,\mu,\nu) = \frac{(2\nu+1)(n+m)!(\nu-\mu)!}{2n(n+1)(n-m)!(\nu+\mu)!}\exp[i(\mu-m)\phi_{lj}]$$

$$\times \sum_{q=1}^{Q_{max}} i^{p+1}\{[(p+1)^2-(n-\nu)^2][(n+\nu+1)^2-(p+1)^2]\}^{\frac{1}{2}}$$

$$\times b(-m,n,\mu,\nu,p+1,p)P_{p+1}^{\mu-m}(\cos\theta_{lj}) \qquad (17)$$

Similarly, $C_{mn\mu\nu}^{j,l}$ and $D_{mn\mu\nu}^{j,l}$ coefficients are obtained by taking the complex conjugate of $\xi(m,n,\mu,\nu)$ and $\zeta(m,n,\mu,\nu)$ from $A_{mn\mu\nu}^{j,l}$ and $B_{mn\mu\nu}^{j,l}$. Here k is propagation constant, $a_q = a(-m,n,\mu,\nu,p)$, $q = 1,2.....,q_{max}$, $p = n+\nu-2q$ and

$$q_{max} = min\left(n,\nu,\frac{n+\nu-|m+\mu|}{2}\right) \qquad (18)$$

Gaunt coefficients have been described in the literature [6]. Hence the total electric field for the l_{th} coordinate system (r,θ,ϕ) may be explicitly written as:

$$\begin{aligned}\mathbf{E}_{total}(l) = \sum_{\nu=1}^{+\infty}\sum_{\mu=-\nu}^{\nu} & \\ & a_{\mu\nu}\mathbf{M}_{\mu\nu}^{(1)}(l)+b_{\mu\nu}\mathbf{N}_{\mu\nu}^{(1)}(l)+e_{\mu\nu}\mathbf{M}_{\mu\nu}^{(3)}(l)+f_{\mu\nu}\mathbf{N}_{\mu\nu}^{(3)}(l)\\ & +\sum_{\nu=1}^{+\infty}\sum_{\mu=-\nu}^{\nu}\sum_{n=1}^{\infty}\sum_{m=-n}^{n}\\ & g_{\mu\nu}\left\{A_{mn}^{\mu\nu}(l,j)\mathbf{M}_{mn}^{(1)}(l)+B_{mn}^{\mu\nu}(l,j)\mathbf{N}_{mn}^{(1)}(l)\right\}+\\ & h_{\mu\nu}\left\{B_{mn}^{\mu\nu}(l,j)\mathbf{M}_{mn}^{(1)}(l)+A_{mn}^{\mu\nu}(l,j)\mathbf{N}_{mn}^{(1)}(l)\right\} \qquad (19)\end{aligned}$$

Similarly the total electric field for the jth coordinate system (r',θ',ϕ') may be explicitly written as:

$$\begin{aligned}\mathbf{E}_{total}(j) = \sum_{\nu=1}^{+\infty}\sum_{\mu=-\nu}^{\nu} & \\ & a_{\mu\nu}^{*}\mathbf{M}_{\mu\nu}^{(1)}(j)+b_{\mu\nu}^{*}\mathbf{N}_{\mu\nu}^{(1)}(j)+g_{\mu\nu}\mathbf{M}_{\mu\nu}^{(3)}(j)+h_{\mu\nu}\mathbf{N}_{\mu\nu}^{(3)}(j)\\ & +\sum_{\nu=1}^{+\infty}\sum_{\mu=-\nu}^{\nu}\sum_{n=1}^{\infty}\sum_{m=-n}^{n}\\ & e_{\mu\nu}\left\{C_{mn}^{\mu\nu}(j,l)\mathbf{M}_{mn}^{(1)}(j)+D_{mn}^{\mu\nu}(j,l)\mathbf{N}_{mn}^{(1)}(j)\right\}+\\ & f_{\mu\nu}\left\{D_{mn}^{\mu\nu}(j,l)\mathbf{M}_{mn}^{(1)}(j)+C_{mn}^{\mu\nu}(j,l)\mathbf{N}_{mn}^{(1)}(j)\right\} \qquad (20)\end{aligned}$$

Applying boundary condition, the tangential component of electric field must continue on the surface of spheres. Using orthogonality properties, following simultaneous linear equations may be described as:

$$e_{\mu\nu} = X_n(a)\left\{a_{\mu\nu}+\left(g_{\mu\nu}\sum_{n=1}^{\infty}\sum_{m=-n}^{n}A_{mn}^{\mu\nu}+h_{\mu\nu}\sum_{n=1}^{\infty}\sum_{m=-n}^{n}B_{mn}^{\mu\nu}\right)\right\}$$

$$f_{\mu\nu} = Y_n(a)\left\{b_{\mu\nu}+\left(g_{\mu\nu}\sum_{n=1}^{\infty}\sum_{m=-n}^{n}B_{mn}^{\mu\nu}+h_{\mu\nu}\sum_{n=1}^{\infty}\sum_{m=-n}^{n}A_{mn}^{\mu\nu}\right)\right\}$$

$$g_{\mu\nu} = X_n(c)\left\{a_{\mu\nu}^{*}+\left(e_{\mu\nu}\sum_{n=1}^{\infty}\sum_{m=-n}^{n}C_{mn}^{\mu\nu}+f_{\mu\nu}\sum_{n=1}^{\infty}\sum_{m=-n}^{n}D_{mn}^{\mu\nu}\right)\right\}$$

$$h_{\mu\nu} = Y_n(c)\left\{b_{\mu\nu}^{*}+\left(e_{\mu\nu}\sum_{n=1}^{\infty}\sum_{m=-n}^{n}D_{mn}^{\mu\nu}+f_{\mu\nu}\sum_{n=1}^{\infty}\sum_{m=-n}^{n}C_{mn}^{\mu\nu}\right)\right\}$$

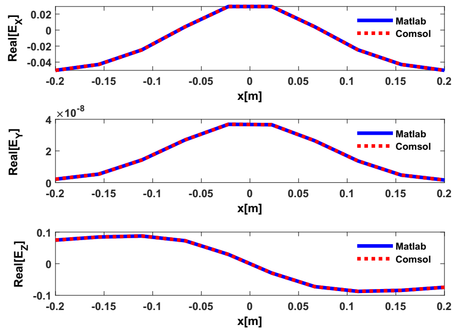

Figure 2. Real part of scattering field from a PEC sphere (blue solid line (Matlab results) and red dot line (COMSOL results)) at zero translation distance.

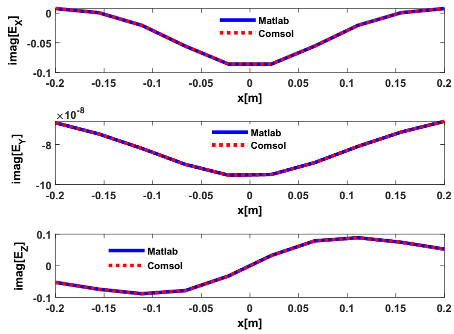

Figure 3. Imaginary part of scattering field from a PEC sphere (blue solid line (Matlab results) and red dot line (COMSOL results)) at zero translation distance.

where

$$X(r_0) = -\frac{j_\nu(kr)}{h_\nu^{(1)}(kr)}|_{r=r_0} \qquad (21)$$

$$Y(r_0) = -\frac{j_\nu'(kr)}{h_\nu'^{(1)}(kr)}|_{r=r_0} \qquad (22)$$

7 Numerical Results and Discussion

In this paper, we reported the fantastic approach of scattering by two PEC spheres. Although this manuscript is basically dedicated to a scattering theory using translation addition theorem for spherical harmonics. We specifically discuss the calculation of the translation coefficients for two PEC spheres. We described preliminary numerical results of the scattered electric field between two PEC sphere in the near zone region. We investigated the scattering behavior, when PEC sphere-I is moving along $z-axis$ with respect to the other. For this purpose, we have been used Eq. 5-10 for numerical simulation using Matlab code.

To clarify the validity of the numerical simulation, we first

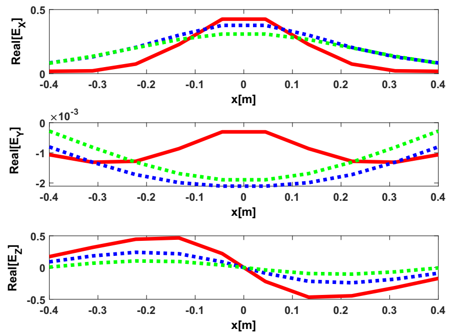

Figure 4. Real part of scattering field from a sphere-I translated along $z-axis$ at $z = 0.4$ m (red solid line), $z = 0.5$ m (blue dot line) and $z = 0.6$ m (green dot line). Here, radius of sphere-I $a = 0.2$ m, radius of sphere-II $c = 0.1$ m

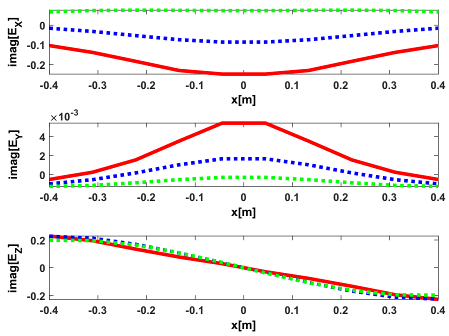

Figure 5. Imaginary part of scattering field from a sphere-I translated along $z-axis$ at $z = 0.4$ m (red solid line), $z = 0.5$ m (blue dot line) and $z = 0.6$ m (green dot line). Here, radius of sphere-I $a = 0.2$ m, radius of sphere-II $c = 0.1$ m

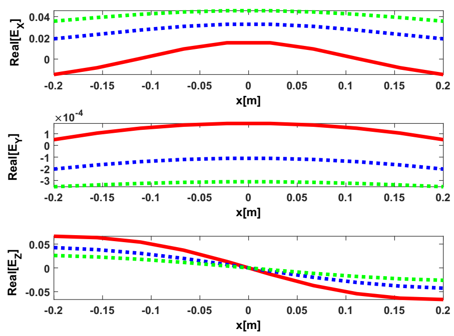

Figure 6. Real part of scattering field from a sphere-I translated along $z-axis$ at $z = 0.4$ m (red solid line), $z = 0.5$ m (blue dot line) and $z = 0.6$ m (green dot line). Here, radius of sphere-I $a = 0.1$ m, radius of sphere-II $c = 0.1$ m

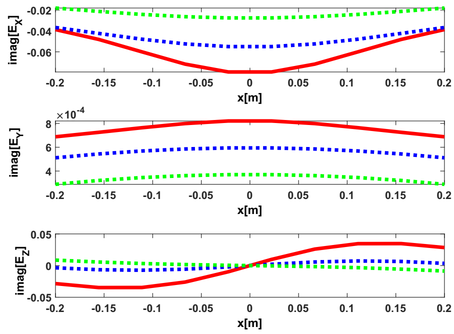

Figure 7. Imaginary part of scattering field from a sphere-I translated along $z-axis$ at $z = 0.4$ m (red solid line), $z = 0.5$ m (blue dot line) and $z = 0.6$ m (green dot line). Here, radius of sphere-I $a = 0.1$ m, radius of sphere-II $c = 0.1$ m

considered overlapping between the two different frames of reference in a cartesian coordinate system. In simple words, we have removed the translation distance between two spheres. Hence, we have reduced our problem from two PEC sphere to one PEC sphere present in free space. During our numerical test: we have taken following parameters, radius of the sphere $a = 0.1$ m, frequency $f = 0.3$ GHz, angle of incident θ_i=0 rad, ϕ_i=1 rad. Suppose the position of cartesian coordinate system can be defined as: field measuring line segment to twice the radius of sphere lies parallel along $x-axis$, that is placed at hight $z = 0.2$ m, y=0; center of the system $x_c = y_c = z_c = 0$; center of the sphere $x_i = 0, y_i = 0, z_i = 0$. The numerical results are accurately matches with the (COMSOL Multiphysics 5.4) shown in Figs. 2, 3.

Further we have decided to translate sphere-I along $z-axis$ with respect to sphere-II. For that, we have chosen parameters: radius of sphere-I $a = 0.2$ m, radius of sphere-II $c = 0.1$ m, frequency $f = 0.3$ GHz, angle of incident θ_i=0 rad, ϕ_i=1 rad, Two frames of references position: center of sphere-I, which is mutually coincided with scattered electric field line distributed parallel along $x-axis$, that is placed at hight $z = 0.4$ m, 0.5 m, 0.6 m and $y = 0$, center of the system $x_c = y_c = z_c = 0$; center of the sphere-II $x_i = 0, y_i = 0, z_i = 0.1$ m. Figs. 4, 5, computed the scattering behavior: when sphere-I is moving away from sphere-II, as is evident from the figures, the real part of scattering electric field of sphere-I is corresponding to the lower numerical results across the translation, and reverse response has been noticed for the case of imaginary part.

When we have considered same radius of both PEC spheres for example: $a = 0.1$ m and $c = 0.1$ m. Figs. 6, 7, presents the scattering performance: when sphere-I is moving away from sphere-II, for the case of real and imaginary numerical results shows higher concentration of scattering electric field. Actually, we have translated sphere along $z-axis$, as consequence of this only real E_z component shows reverse response, graphical representation show lower concentration of scattering field with respect to the translation of a sphere-I.

8 Conclusion

We study multiple scattering electric field pattern using two PEC sphere. We derived mathematical formation of scattered field using vector translation AT for spherical harmonics. We visualized the scattering electric field using computational approach. When we have been translated sphere-I with respect to sphere-II with help of simulation code. The numerical results of scattering field were leading or somewhere lagging across the movement of sphere-I along $z-axis$.

References

[1] Batool S., Nisar M., Mangini F., Frezza F., Fazio E., "Scattering of Light from the Systemic Circulatory System", *Diagnostics* **10**, 2020 1026.

[2] Batool S., Nisar M., Mangini F., Frezza F., Fazio E., "Polarization Imaging for Identifying the Microscopical Orientation of Biological Structures", *In Proceedings of the URSI GASS Conference*, Rome, Italy 2020 Aug (Vol. 29).

[3] Fazio E., Batool S., Nisar M., Mangini F., Frezza F., "Recognition of bio-structural anisotropy by polarization-resolved imaging", *Biophysical Review*(submitted).

[4] Lagendijk A., Van Tiggelen B. A., "Resonant multiple scattering of light", *Physics Reports*, **270**, 1996 143-215.

[5] Batool S., Frezza F., Mangini F., Simeoni P., "Introduction of Radar Scattering Application in Remote Sensing and Diagnostics". *Atmosphere*. **11**, 2020 517.

[6] Batool S., Frezza F., Mangini F., Yu-Lin X., "Scattering from multiple PEC sphere using translation addition theorems for spherical vector wave function", *Journal of Quantitative Spectroscopy and Radiative Transfer*, **248**, 2020 106905.

[7] Batool S., Benodetti A., Frezza F., Mangini F., Xu YL, "Effect of finite terms on the truncation error of addition theorems for spherical vector wave function", In 2019 PhotonIcs and Electromagnetics Research Symposium-Spring (PIERS-Spring) 2019 Jun 17, 2795-2801.

[8] Batool S., Naqvi Q. A., Fiaz M. A., "Scattering from a cylindrical obstacle deeply buried beneath a planar non-integer dimensional dielectric slab using Kobayashi potential method". *Optik-International Journal for Light and Electron Optics*, **153**, 218 95-108.

[9] Saillard M., Sentenac A., "Rigorous solutions for electromagnetic scattering from rough surfaces", *Waves in Random Media*, **11**, 2001 103-37.

[10] Liang C., Lo Y. T., "Scattering by two spheres", *Radio Science*, **12**, 1967 1481-95.

[11] Mackowski D. W., "Analysis of radiative scattering for multiple sphere configurations", *Proceedings of the Royal Society of London. Series A: Mathematical and Physical Sciences*, **433**, 1991 599-614.

[12] Xu Y. L., "Electromagnetic scattering by an aggregate of spheres", *Applied Optics*, **34** 1995, 4573-4588.

[13] Friedman B., Russek J., "Addition theorems for spherical waves", *Quarterly of Applied Mathematics*, **1**, 1954 13-23.

[14] Stein S., "Addition theorems for spherical wave functions", *Quarterly of Applied Mathematics*, **19**, 1961 15-24.

[15] Cruzan O. R., "Translational addition theorems for spherical vector wave functions", *Quarterly of Applied Mathematics*, **20**, 1962 33-40.