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Efficient Modeling of Radio Wave Propagation in Tunnels for 5G and Beyond Using a Split-Step Parabolic Equation Method

Hao Qin and Xingqi Zhang
University College Dublin, Belfield, Ireland
hao.qin@ucdconnect.ie, xingqi.zhang@ucd.ie

Abstract

This paper presents a three-dimensional radio wave propagation model that is suitable for 5G and beyond frequency bands in tunnels. The model is based on a split-step parabolic equation (SSPE) method, which can achieve superior performance at high frequencies compared to the widely used finite-difference parabolic equation (FDPE) method. The accuracy and efficiency of SSPE-based models are demonstrated through comparisons with analytical solutions as well as FDPE-based models in rectangular waveguides. Numerical results are also validated against experimental measurements in a realistic tunnel geometry.

1 Introduction

As the demand for high-quality communications for train operation as well as passengers' experience continues to grow, the advent of 5G and beyond wireless communication systems is highly sought after in the railway sector. The study of radio wave propagation characteristics at such frequency bands is essential for the effective design and deployment of corresponding communication systems.

In recent years, various modeling techniques for radio wave propagation in railway environments have been presented in the literature. Among them, two popular methods that have been extensively used are the ray-tracing and parabolic equation methods [1–5]. The ray-tracing method can be applied to solve radio wave propagation in complex environments such as railway stations. However, in enclosed structures like tunnels, the presence of multiple reflections off the walls compromises the efficiency of ray-tracing. The parabolic equation method, on the other hand, can be applied to long guiding structures with high accuracy and efficiency. However, the model that is currently widely used for tunnel propagation studies is based on a finite-difference approach. The computational cost of the finite-difference parabolic equation (FDPE) method grows significantly as the frequency increases, which makes it less suitable for radio wave characterization at 5G and beyond.

In this paper, a three-dimensional radio wave propagation model, based on a split-step Fourier technique, is presented. This model offers is a more computationally efficient solution for radio wave prorogation studies at high frequencies. Numerical results are validated against FDPE-based simulation models and measurements in the Massif Central tunnel in south-central France.

2 Split-Step Parabolic Equation Method

The derivation of the parabolic equation method starts with the Helmholtz wave equation for free-space:

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}+{k_0}^2\right)\phi(x,y,z)=0 \tag{1}$$

where k_0 is the free-space wavenumber, and ϕ denotes the scalar electric or magnetic potential. Assuming propagation predominantly along the z-axis, the standard parabolic equation can be expressed as:

$$\frac{\partial u}{\partial z}=\frac{1}{2jk_0}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)u \tag{2}$$

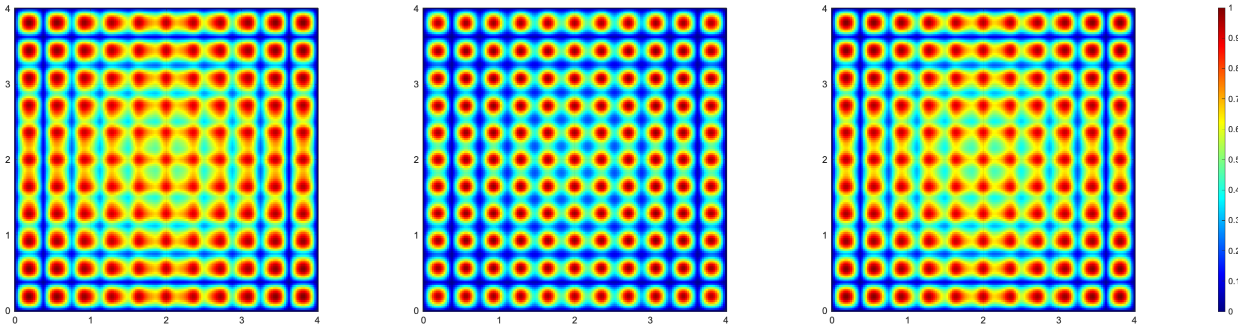
where u is the reduced plane wave solution. A vectorial form of (2), referred to as the vector parabolic equation, can be used to determine all field components and address the coupling effects [6]. Generally, finite-difference based schemes, such as the Crank-Nicolson and the Alternating-Direction-Implicit methods [7–9], are applied for the numerical implementation of (2). However, small discretization steps are required at high frequencies, leading to a large computational cost for the simulation.

Alternatively, (2) can be numerically implemented using a split-step Fourier technique [10], refer to as the split-step parabolic equation (SSPE) method. The SSPE method uses a longitudinally marching procedure with the help of discrete Fourier transform. The antenna pattern is projected to the initial two-dimensional transverse plane. Then, a new profile is obtained based on fields from the previous plane:

$$u(x,y,z+\Delta z)=F^{-1}\left\{C(k_x)\,C(k_y)\,F\{u(x,y,z)\}\right\} \tag{3}$$

where F and F^{-1} are the Fourier transform pair; k_x and k_y are the spectral variables, and

$$C(k)=\exp\left(\frac{-jk^2\Delta z}{2k_0}\right) \tag{4}$$



(a) (b) (c)

Figure 1. The spatial distribution of the analytical and simulated fields on the transverse plane at a distance of 50 m from the initial plane in a rectangular waveguide. (a) Analytical solution. (b) FDPE. (c) SSPE.

The procedure is repeated until the propagator reaches the desired range.

3 Numerical Examples

In this section, the accuracy and efficiency of SSPE-based models are demonstrated through comparisons with analytical solutions as well as FDPE-based models in a rectangular waveguide.

The cross-section dimensions of the rectangular waveguide are 4 m×4 m, and the simulated distance is 50 m. A unit-strength Gaussian source at 10 GHz is placed at the center of the transverse plane as the initial source. The spatial discretization steps on the transverse plane are chosen as $\Delta x = \Delta y = 0.8\lambda$, and the spatial discretization step along the propagation direction is chosen as $\Delta z = 8\lambda$. The simulated results obtained from the SSPE- and FDPE-based models are compared to the analytical solutions [10] through a Euclidean error norm:

$$E_{\text{rms}} = \frac{\sqrt{\frac{1}{N}\sum_i\sum_j \left| u_{i,j}^{\text{numerical}} - u_{i,j}^{\text{analytical}} \right|^2}}{\sqrt{\frac{1}{N}\sum_i\sum_j \left| u_{i,j}^{\text{analytical}} \right|^2}} \tag{5}$$

where N is the total number of sampling points; $u_{i,j}^{\text{numerical}}$ and $u_{i,j}^{\text{analytical}}$ denote numerically and analytically computed fields at the (i, j)-th sampling point, respectively.

In Fig. 1, the spatial distribution of the analytical and simulated fields on the transverse plane at a distance of 50 m from the initial plane is compared. The relative error of the SSPE- and FDPE-based models are 3 % and 19 %, respectively. It can observed that SSPE has a better performance than FDPE for the same simulation setup.

Then, we fix the relative error at 2 %, and plot the CPU time required for each propagation model as a function of the simulated frequencies. The comparison is shown in Fig. 2, where SSPE has a superior performance than FDPE, especially at high frequencies.

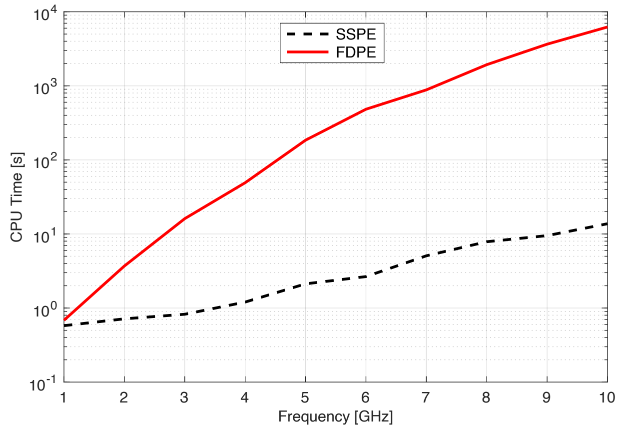
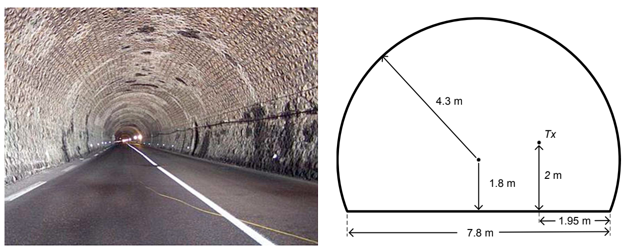


Figure 2. Comparison of the CPU time of the SSPE and FDPE simulations at different frequencies.

4 Application: Massif Central Tunnel

In this session, the SSPE method is applied to a tunnel geometry that has been widely studied in the literature: the Massif Central tunnel in south-central France [11]. It is a straight, long tunnel, as shown in Fig. 3 (a). In the simula-



(a) (b)

Figure 3. The geometry of the Massif Central tunnel. (a) On-site image [12]. (b) Cross-section geometry.

tion, the electrical parameters of the surrounding walls are set to $\varepsilon_r = 5$ and $\sigma_0 = 0.01$ S/m. A vertically polarized horn antenna operating at 10 GHz is employed. Both the transmitter and the receiver are located at a height of $y = 2$ m and a horizontal offset of one quarter of the width of the tunnel, as illustrated in Fig. 3 (b). The whole simulated distance is 2.5 km.

The received power generated by the SSPE method is verified against measured data, as shown in Fig. 4. It can be observed that the profiles of the received power are in good agreement.

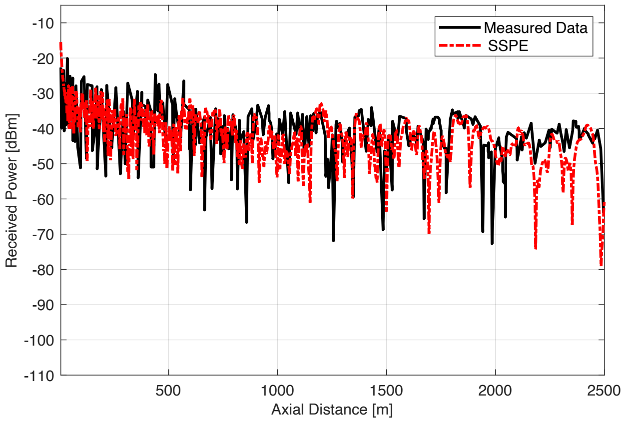


Figure 4. Received power at 10 GHz in the Massif Central tunnel.

5 Conclusion

In this paper, a three-dimensional SSPE-based model was presented to characterize radio wave propagation in tunnels. The model has shown superior performance over the widely used FDPE-based models, especially at high frequencies. As new-generation systems operating at 5G and beyond frequencies are being deployed for railway applications, the SSPE-based model presented in this paper offers a promising solution for the accurate and efficient characterization of corresponding wireless channels.

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