Higher-order harmonics scattering cancellation by thin metasurfaces for dielectric cylinders

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Abstract

We present a closed-form expression for the impedance of a metasurface coating for a dielectric cylinder, able to cancel all m-indexed harmonic scattered by the cylinder until an arbitrary order M. Combining Mie theory with the transmission line methodology, a dielectric scatterer, seen as a generic load admittance Y_a, is covered by a mantle cloak with surface admittance function $Y_s(M,\phi)$, depending on the M index and on the azimuthal variable ϕ. It is demonstrated that the scattered energy is hidden inside the dielectric object, forming internal nonradiating modes supported by the azimuthally varying complex surface impedance boundary.

1 Scattering reduction for cylinders

Reducing scattering from a metallic or dielectric cylinder is a classical problem in electromagnetics [1]-[3]. One possibility to do it is to re-route the electromagnetic field around the obstacle by means of inhomogeneous and anisotropic cloaking shells as designed by transformation optics (TO) [4]-[6] or let the waves pass through the object-cloak pair without any distortion as predicted by scattering cancellation approaches [7, 8]. In the quasi-static regime, where the object is very small compared to the incident wavelength, it is sufficient to enforce a single-dominant harmonic cancellation with a single volumetric metamaterial coating [7] or by a single thin impedance metasurface [8]. The last problem of coating an object with the mantle cloaking reduces to a lumped impedance matching problem [9], by invoking a network formalism of the pertinent Green's function. For this approach, a single azimuthally constant cloak has the role of reducing the *mismatch* between the dielectric and free space, seen as dispersive lumped loads in the near-field by each independent single harmonic wave [9]. As represented in Fig. 1, when the scattering is dominated by more cylindrical harmonic waves (modeled by Hankel functions), a simultaneous m-index cancellation is necessary: in quasi-static regime, cylindrical scattering is reduced to only $m=0$ (the lowest order harmonic wave), but in other frequency regimes cancellation of several higher-order harmonic waves can surprisingly be valid [10]. However, single-harmonic suppression is no more enough beyond subwavelength size and an advanced multi-harmonics mantle cloak is needed. A balanced loss-gain metasurface has been found to appear as a result of enforcing all the scattering coefficients to be zero for a metallic cylinder [11]. In this paper, we propose a general methodology for cloaking a given set M of independent harmonics. This will be performed in a more general framework considering a two-layer dielectric cylinder (seen as generic admittance loads) covered by an impedance metasurface at a certain distance.

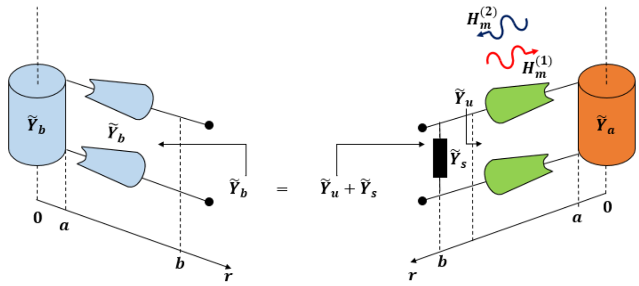

Figure 1. Cloaking as matching for dielectric cylinders. Complete background scenario, modeled as lumped input admittance (all the values, tilde sign on top, are here intended as normalized) $\widetilde{Y}_b$ (left) and cloaked device, made up of generic load admittance $\widetilde{Y}_a$, traveling in a cylindrical transmission line (overall input admittance $\widetilde{Y}_u$), and an inserted surface admittance $\widetilde{Y}_s$.

2 Zero scattered fields in Mie Theory

Consider a dielectric cylinder of radius a, loaded by an impedance sheet at the radius $b=a+d$, where d can be the thickness of a certain dielectric substrate. The overall scattering, according to Mie Theory [8] is given by

$$\mathrm{E}_z(r,\phi)=\begin{cases}\sum_{-\infty}^{\infty} j^{-m}\,\mathrm{e}_m(k_a r)\,e^{jm\phi}, & r\le a\\ \sum_{-\infty}^{\infty} j^{-m}\,\mathrm{f}_m(k_{ab} r)\,e^{jm\phi}, & a\le r\le b\\ \sum_{-\infty}^{\infty} j^{-m}\,\mathrm{g}_m(k_0 r)\,e^{jm\phi}, & r\ge b\end{cases} \quad (1)$$

where the magnetic fields can be derived via the radial derivatives, whereas $k_0=\omega\sqrt{\varepsilon_0\mu_0}$, $k_{ab}=\sqrt{\varepsilon_{ab}}k_0$ and $k_a=\sqrt{\varepsilon_a}k_0$ are the wavenumbers in the background free-space region, in the annulus shell (with relative permittivity ε_{ab}, normalized to the background permittivity) and in the dielectric cylinder (with relative permittivity ε_r), respectively.

All computations are performed according to the $e^{+j\omega t}$ time convention, as assumed and suppressed throughout this work. The radial functions $e_m(\cdot)$, $f_m(\cdot)$ and $g_m(\cdot)$ contains unknown expansion coefficients and are related to the cylindrical wave harmonics as

$$\mathrm{e}_m(k_a r) = a_m J_m(k_a r) \tag{2}$$

$$\mathrm{f}_m(k_{ab} r) = b_m H_m^{(1)}(k_{ab} r) + d_m H_m^{(2)}(k_{ab} r) \tag{3}$$

$$\mathrm{g}_m(k_0 r) = J_m(k_0 r) + c_m H_m^{(2)}(k_0 r) \tag{4}$$

where $J_m(\cdot)$ is the Bessel function, whereas $H_m^{(1)}(\cdot)$ and $H_m^{(2)}(\cdot)$ are the ingoing (first kind) and outgoing (second kind) Hankel functions, respectively (Fig. 1). Once applied the tangential boundary conditions, the unknown expansion coefficients a_m, b_m, d_m and c_m are obtained. The boundary conditions are imposed in terms of discontinuity of the total magnetic field as

$$Y_s\, \mathrm{E}_z(b^-,\phi) = \left[\mathrm{H}_\phi(b^+,\phi) - \mathrm{H}_\phi(b^-,\phi)\right] \tag{5}$$

where Y_s is the surface admittance covering the external surface at $r = b$. Eq. (5) can be rewritten as

$$Y_s(\phi) = \left[\frac{\mathrm{H}_\phi(b^+,\phi)}{\mathrm{E}_z(b,\phi)} - \frac{\mathrm{H}_\phi(b^-,\phi)}{\mathrm{E}_z(b,\phi)}\right] = Y_b(\phi) - Y_u(\phi) \tag{6}$$

where now the surface admittance $Y_s(\cdot)$ is seen as the residual difference between two admittance functions: the background admittance Y_b in region at $r = b^+$ as shown in Fig. 1 (left) and the uncloaked admittance Y_u seen from the region at $r = b^-$ in Fig. 1 (right). It is worthwhile mentioning that they are all explicit functions of the azimuthal angle ϕ. When the cloaking condition $c_m = 0$ is inserted (ideally for any m) and the fields within the dielectric cylinder are computed accordingly, Eq. (6) can be interpreted as a cylindrical matching formula which ensures no discontinuity between the background admittance Y_b and the parallel between the uncloaked and surface admittances $(Y_u + Y_s)$. In the following, the admittance values will be considered as normalized with respect to $Y_B = k_0(\omega\mu_0)^{-1} = \sqrt{\varepsilon_0/\mu_0}$, the intrinsic admittance of the background. As inspired by [11], the cylindrical harmonics are not considered singularly, one-by-one, but all simultaneously with their azimuthal weights. The reference vacuum admittance, as ratio of magnetic field over electric field, is computed as

$$\widetilde{Y}_b(\phi) \equiv \frac{Y_b(\phi)}{Y_B} = -j\frac{\sum_{-\infty}^{\infty} j^{-m}\, J'_m(k_0 b)\, e^{jm\phi}}{\sum_{-\infty}^{\infty} j^{-m}\, J_m(k_0 b)\, e^{jm\phi}} \tag{7}$$

The second term in Eq. (6), representing the uncloaked admittance, can be computed as

$$\widetilde{Y}_u(\phi) \equiv \frac{Y_u(\phi)}{Y_B} = \frac{1}{Y_B}\frac{\mathrm{H}_\phi(b^-,\phi)}{\mathrm{E}_z(b^-,\phi)} = \tag{8}$$

$$= -j\sqrt{\varepsilon_{ab}}\frac{\sum_{-\infty}^{\infty} j^{-m}\left[b_m H_m^{(1)'}(k_{ab} b) + d_m H_m^{(2)'}(k_{ab} b)\right] e^{jm\phi}}{\sum_{-\infty}^{\infty} j^{-m}\, J_m(k_0 b)\, e^{jm\phi}}$$

where the continuity of the tangential electric field is exploited. The unknown coeffients can be solved for $\gamma_m \equiv b_m/d_m$, giving

$$\gamma_m(\widetilde{Y}_a) \equiv \frac{b_m}{d_m} = -\frac{\widetilde{Y}_a H_m^{(2)}(k_{ab}a) + jH_m^{(2)'}(k_{ab}a)}{\widetilde{Y}_a H_m^{(1)}(k_{ab}a) + jH_m^{(1)'}(k_{ab}a)} \tag{9}$$

where the uncloaked object is seen as a normalized load admittance $\widetilde{Y}_a$ with value

$$\widetilde{Y}_a \equiv -j\sqrt{\frac{\varepsilon_a}{\varepsilon_{ab}}}\frac{J'_m(k_a a)}{J_m(k_a a)} \tag{10}$$

All the normalizations, including the surface admittance function, are performed with respect to $Y_B = \sqrt{\varepsilon_0/\mu_0}$. The interpretation of the $\widetilde{Y}_a$ is the normalized input admittance of a dielectric cylinder of radius a and relative permittivity ε_a as seen from a transmission line having a medium with ε_{ab} as relative permittivity. In addition, the physical interpretation of the coefficient γ_m is taking into account the fraction of field (electric or magnetic) reflected (b_m) for any (electric or magnetic) transmitted field (d_m). The final formula can be compactly written as

$$\widetilde{Y}_s(M,\phi) = -j\frac{\sum_{m=-M}^{m=+M} j^{-m}\, \Delta_m\, J_m(k_0 b)\, e^{jm\phi}}{\sum_{m=-M}^{m=+M} j^{-m}\, J_m(k_0 b)\, e^{jm\phi}} \tag{11}$$

where

$$\Delta_m = \widetilde{B}_m(k_0 b) - \widetilde{U}_m(k_{ab} b, \widetilde{Y}_a) \tag{12}$$

$$\widetilde{B}_m = \frac{J'_m(k_0 b)}{J_m(k_0 b)} \tag{13}$$

$$\widetilde{U}_m = \sqrt{\varepsilon_{ab}}\frac{H_m^{(2)'}(k_{ab} b) + \gamma(\widetilde{Y}_a) H_m^{(1)'}(k_{ab} b)}{H_m^{(2)}(k_{ab} b) + \gamma(\widetilde{Y}_a) H_m^{(1)}(k_{ab} b)} \tag{14}$$

It is worthwhile mentioning that now the series has been truncated to a finite value M, which represents the number of harmonic waves that is possible to compensate in the design: the ideal case of zero scattering reduction is obtained for the limit $M \to \infty$. Eq. (11) has been derived in a general condition: cloaking a generic normalized load $\widetilde{Y}_a$, wrapped by a dielectric substrate with relative permittivity ε_{ab} at $r = a$, loaded with a surface admittance at $r = b$. The surface admittance function is highly dependent on Δ_m, which represents a residual term needed to cancel scattered fields for a certain number M of harmonic waves. For single harmonic suppression, Eq. (11) gets simplified and the condition reduces to

$$\widetilde{Y}_s(m = n) \equiv -j\Delta_{m=n} = -j\frac{J'_n(k_0 a)}{J_n(k_0 a)} - \widetilde{Y}_a(k_a a) \tag{15}$$

which is consistent with the condition derived in[9, 12]. As stressed above, this shows that any single-harmonic cancellation for any arbitrary index $m = n$ (not only for the lowest order mode) gives a surface impedance value which is purely reactive.

3 Numerical Results

We will show numerical results for a dielectric cylinder with relative permittivity $\varepsilon_a = 3$ and overall diameter $2a = \lambda_0$. For simplicity, the mantle cloak studied here is directly attached to the object's surface and it is described by a complex surface impedance function $Z_s(\phi) = R_s(\phi) + jX_s(\phi)$ obtained by inverting and denormalizing the surface admittance $\widetilde{Y}_s$, for a given set of harmonic components M. Fig.

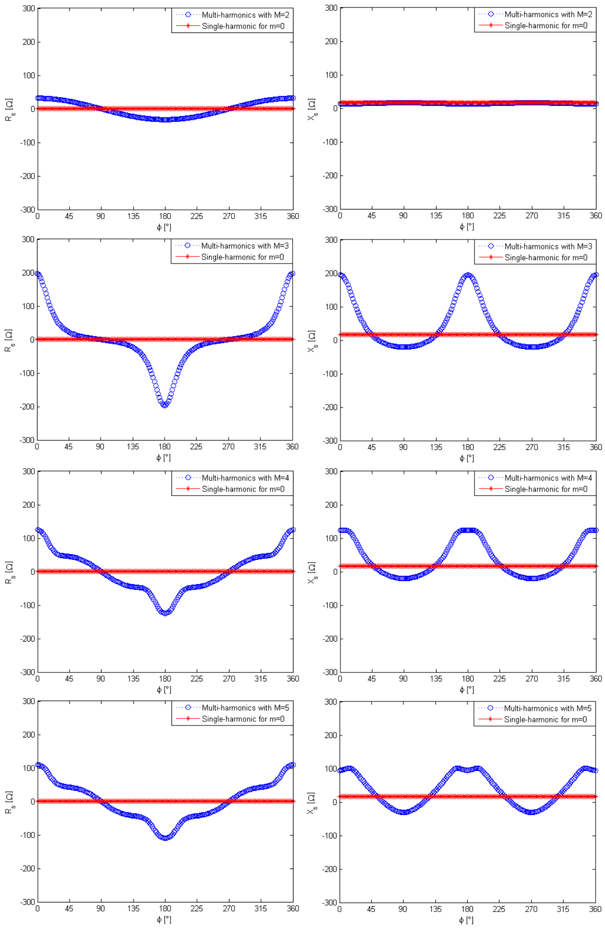

Figure 2. Real (left column) and imaginary parts (right column) of the surface impedance $Z_s(\phi) = R_s(\phi) + jX_s(\phi)$. The individual rows are relevant to suppress from $M = 1$ or $M = 2$ (same plot, top) towards $M = 3$, $M = 4$ and $M = 5$ (bottom).

2 shows the real part (R_s, left column) and the imaginary part (X_s, right column) of the surface impedance. Also reported, it is a reference surface impedance value for single-harmonic cancellation (the lowest order mode $m = 0$). As expected, it is purely imaginary ($R_{s_0} = 0$) and with a specific inductive behaviour ($X_{s_0} = +16.35\ \Omega$) to compensate for the capacitive effect of the dielectric cylinder. A complex oscillating behaviour (left column of Fig. 2) with a zero mean value is obtained (note that the incident direction is at $\phi = 0°$). The incoming wave encounters first the positive-R_s resistance (lossy side), located in front at $-90° < \phi < +90°$, whereas the negative-R_s resistance (gain side) is placed behind the object at $+90° < \phi < +270°$. The effect of summing more and more harmonics obviously changes the behaviour of the metasurface. However, a con-

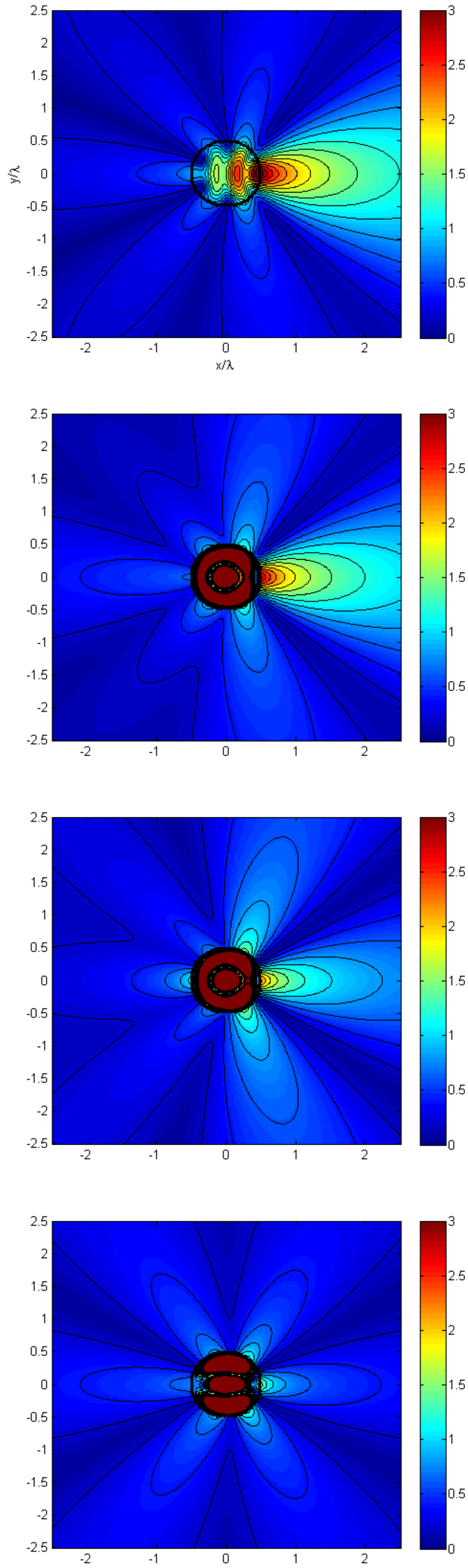

Figure 3. Amplitude of the scattered electric field for a dielectric object with radius $a = 0.5\lambda_0$ and relative permittivity $\varepsilon_r = 3$. From top to bottom: no metasurface coating and multi-harmonics cloaking metasurface with $M = 1, 2, 3$, respectively.

vergence trend is noticed for increasing M. The scattered field corresponding to the same case of Fig. 2 is shown in Fig. 3 (the impinging wave is coming from $+\hat{x}$). Increasing M in Eq. (11) implies a gradual reduction of the scattered field. The uncloaked cylinder reported at the top of the column in Fig. 3 should be considered the reference case for looking at the scattering reductions. The insertion of a mantle cloak gradually reduces the forward and backward scattering beams for increasing M. The scattered energy that would naturally radiate outside is kept within the device itself, with self-trapped energy that starts increasing, within the cloaked object. A nonradiating mode is formed and it is sustained y this complex oscillating impedance boundary. With the last case having $M = 3$, the cloaking system has almost no scattering energy outside its domain of definition and the formed internal nonradiating modes have some similarities with the radiationless anapole mode in dielectric particles [13]. Implementations of these equivalent complex surface impedance functions with lossless metasurface platforms are currently under investigation.

4 Conclusions

In this paper, complex surface impedance functions have been obtained in order to suppress M harmonics scattered by a bare dielectric cylinder. A combination of Mie theory and transmission line formalism has been used. The final expression leads to a non uniform loss-gain metasurface globally neutral, whose behaviour depends by the number M of harmonics to be canceled, but it converges to a specific limit for an infinite number of harmonics. It is seen that the cloaking metasurface supports an increasing non-radiating energy inside the cloaked region for increasing M.

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