Relative Stability of First-Order and Second-Order Optical Phase-Locked Loop Considering Non-Negligible Propagation Delay

Jayjeet Sarkar
Department of Applied Electronics and Instrumentation Engineering
Academy of Technology
Hooghly-712121, West Bengal, INDIA
e-mail: jayjeetsarkar@gmail.com

Abhijit Banerjee
Department of Electronics and Communication Engineering
Academy of Technology
Hooghly-712121, West Bengal, INDIA
e-mail: abhb7887@yahoo.co.in

B.N.Biswas
Education Division
SKF Group of Institution,
Sir J.C. Bose School of Engineering
Hooghly-712121, West Bengal, INDIA
e-mail: b.baidyanath@yahoo.com

***Abstract*— The effect of loop delay on the relative stability of first- and second-order optical phase-lock loop (OPLL) is analyzed. The gain margin and phase margin for OPLL with modified first-order loop filter and active low-pass loop filter are demonstrated considering all the parameters affecting the loop stability. The tracking performance of OPLL in terms of phase-error variance is highlighted considering shot noise, white frequency induced phase noise and loop propagation delay in stable operating condition, too.**

Keywords*— *Optical phase-lock loop, Gain margin, Phase margin, Phase-crossover frequency and Gain-crossover frequency.

I. INTRODUCTION

OPLL has innumerable applications in laser frequency stabilization, coherent optical communication systems, and low noise microwave or mm-wave generation. In last decade, lot of research has been carried out on OPLL [1]-[6]. OPLL has finite time-delay. The loop-delay affects the phase response of the loop output imposing a restriction on the higher value of the loop natural frequency due to the stability condition [6] and the phase-error variance even at the optimum condition rises sharply with delay. Lowering of the loop natural frequency decreases the pull-in range, increases the pull-in time, tracking error and phase-error variance etc. OPLL should be operated well within the stability region. So phase margin (PM) and gain margin (GM) are the measures of degree of loop stability. The influence of loop-delay on the PM of first- and second-order loops for both the discrete and continuous time variety has already be shown by Bergman [7]. For a modified PSK homodyne optical receiver stability analysis, the impacts of damping factor and loop-gain on the PM has been discussed in [8]. In [9], the maximum limit of loop propagation delay for stable operation of second-order OPLL was reported.

II. THEORY

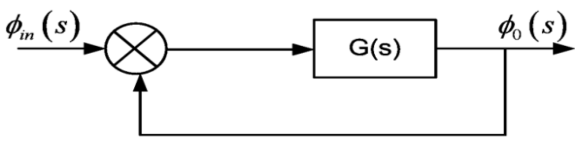

Fig. 1. Block schematic of the OPLL.

Fig.1 shows an OPLL where the local oscillator (LO) laser phase forms the output signal. The open-loop transmission, $G(s)$ is given by the phase-detector gain (K_{PD}), loop filter transfer function ($F(s)$) and LO laser tuning efficiency (K_{VCO}). A first-order loop-gain function contains one integration within the frequency tuning of the LO laser whereas second-order contains another. So, the open-loop transfer function of the loop is given by

$$G(s) = \frac{K}{s} F(s) \exp(-s\tau) \quad (1)$$

where $s(= j\omega)$ is the Laplace-variable, τ is the loop delay (sec), and $K\ (= K_{PD} K_{VCO})$ is the loop-gain in Hz. The transfer function is given by

$$H(s) = \frac{\phi_o(s)}{\phi_{in}(s)} = \frac{G(s)}{1+G(s)}. \quad (2)$$

A. Phase Margin and Gain Margin

The gain margin (GM) is the gain perturbation that makes the system marginally stable and is measured at the phase crossover frequency $\omega = \omega_p$ for which $\angle G(j\omega) = -\pi$, where, ω_p is the phase-crossover frequency [10].

$$GM = 20\log_{10}\frac{1}{\left|G\left(j\omega_p\right)\right|}\ . \quad (3)$$

The phase margin (PM) is the negative phase perturbation that makes the system marginally stable and is determined at the unity-gain frequency of $G(j\omega)$ [10], i.e., at $\omega=\omega_g$ for which $\left|G\left(j\omega\right)\right|=1$, where ω_g is the gain-crossover frequency. If the phase $\phi(\omega)$ of $G\left(j\omega\right)$ exceeds $-\pi$ rad at $\omega=\omega_g$, the system will be unstable By definition,

$$PM = \pi + \phi\left(\omega_g\right). \quad (4)$$

We first consider the first-order OPLL, i.e., $F\left(s\right)=1$. We may write

$$G\left(j\omega\right) = \frac{K}{j\omega}\exp\left(-j\omega\tau\right) \quad (5.1)$$

with $\angle G(j\omega)=-\dfrac{\pi}{2}-\omega\tau$ and $\left|G\left(j\omega\right)\right| = \dfrac{K}{\omega}$. (5.2)

From (5.2), we get, $\omega_p = \dfrac{\pi}{2\tau}$, and $\omega_g = K$. (6)

Using (3), (4) and (6), we get,

$$GM = 20\log_{10}\left(\frac{\pi}{2K\tau}\right) \text{and}\ PM = \left(\frac{\pi}{2}-K\tau\right)\ . \quad (7)$$

Expression (7) shows that both GM and PM decreases as normalized loop delay $d\left(=K\tau\right)$ increases. The edges of the stability region are determined by setting $GM=0$ dB and $PM=0°$. So at the edge of stability, $d=\pi/2$. In practical systems PM of around $\pi/4.5$ to $\pi/3$ rad are required for which d will be $5\pi/18$ to $\pi/6$, respectively; and GM of 10 and 30 dB are obtained for $d=0.497$ and $d=0.049$, respectively.

We next consider the modified first-order loop filter with transfer function [6], $F\left(s\right) = \dfrac{1}{\left(1+sT\right)}$ (8)

where T is the filter time constant. Substituting (8) into (1) we may write

$$G\left(j\omega\right) = \frac{K}{j\omega}\left(\frac{1}{1+j\omega T}\right)\exp\left(-j\omega\tau\right). \quad (9)$$

To calculate the PM, we first calculate $\left|G\left(j\omega\right)\right|^2$ and equate that to 1. After few steps of mathematical simplifications, we get

$$\omega_g = \omega_n F_1\left(\xi\right) \text{ where } F_1\left(\xi\right) = \left[-2\xi^2+\sqrt{4\xi^4+1}\right]^{1/2} \quad (10)$$

where $\omega_n\left(=\sqrt{K/T}\right)$, $\xi\left(=\omega_n/2K\right)$ are the loop-natural frequency and damping coefficient of the OPLL with the modified first-order loop filter, respectively.

Finally, the PM can be expressed

$$PM = \pi+\phi\left(\omega_g\right) = \frac{\pi}{2}-\tan^{-1}\left(\frac{F_1\left(\xi\right)}{2\xi}\right)-dF_1\left(\xi\right) \quad (11)$$

where $d\left(=\omega_n\tau\right)$ is the normalized loop propagation delay.

To calculate the GM of the loop, we first calculate $\angle G\left(j\omega\right)$ and equate that to $-\pi$. After simplifications and approximating the tangent term up to third order, one obtains,

$$\omega_p = \omega_n F_2\left(\xi, d\right)$$

where $F_2\left(\xi, d\right) = \left[\dfrac{-3+\sqrt{24\xi d+9}}{2d^2}\right]^{1/2}$ (12)

and $GM = 20\log_{10}\left[F_2\sqrt{16\xi^4+F_2^2}\right].$ (13)

Considering an active low-pass loop filter with transfer function [1] $F\left(s\right) = \dfrac{1+s\tau_2}{s\tau_1}$ (where τ_1 and τ_2 are the filter time-constants), and using the same procedure stated earlier, for second-order OPLL with loop propagation delay we get

$$\omega_g = \omega_n F_3\left(\xi\right),$$

where $F_3\left(\xi\right) = \left[2\xi^2+\sqrt{4\xi^4+1}\right]^{1/2}$ (14)

and $PM = \tan^{-1}\left(2\xi F_3\left(\xi\right)\right)-dF_3\left(\xi\right)$ (15)

where $\omega_n\left(=\sqrt{K/\tau_1}\right)$, $\xi\left(=\omega_n\tau_2/2\right)$ and $\omega_p = \omega_n F_4\left(\xi, d\right)$

where $F_4\left(\xi, d\right) = \left[\dfrac{3\left(2\xi-d\right)}{d^3}\right]^{1/2}$ (16)

and $GM = 20\log_{10}\left[\dfrac{F_4^2}{\sqrt{4\xi^2F_4^2+1}}\right].$ (17)

The minimum damping factor $\xi_{\min}$ that is required to achieve a prescribed PM of OPLL with modified first-order loop filter and active low-pass loop filter can be obtained from (11) and (14), are given respectively, by,

$$\xi_{\min} = \left[\frac{1}{16\cot^2(PM)(\cot^2(PM)+1)}\right]^{1/4} \quad (18)$$

and $\xi_{\min} = \left[\dfrac{\tan^4(PM)}{16(\tan^2(PM)+1)}\right]^{1/4}.$ (19)

B. Phase-error Variance

The OPLL performance depends on its ability to track the phase of the incoming signal. Any error in tracking degrades the performance, is represented by the phase-error variance. The phase-error variance of the dither OPLL with loop propagation delay is given by [6]

$$\sigma_E^2 = \frac{\Delta\nu}{\pi\omega_n}\int_0^\infty\left|\frac{1-H\left(x\right)}{x}\right|^2 dx+\frac{q\omega_n}{2RP_R}\int_0^\infty\left|H\left(x\right)\right|^2 dx \quad (20)$$

where $\Delta\upsilon$ is the summed laser line-width in Hz, P_R denote the power of received signal in W, q is the electron charge in Coulomb and R is the photo-detector responsivity in A/W. $H(x)$ can be obtained from (1) and (2), and $x = \omega/\omega_n$. The quantities on the right hand side of (20), respectively, denote the variances due to white frequency noise induced laser phase noise and shot noise.

III. RESULTS AND DISCUSSION

Fig. 2 shows GM gradually decreases as normalized loop propagation delay d increases, the OPLL becomes more susceptible to gain perturbations, both for damping coefficient $\xi = 0.707$ and $\xi = 1.0$. For low values of d, the second-order OPLL is slightly less susceptible to gain perturbations than the modified first-order and for higher values of d, the situation is reversed. For modified first- and second-order, d should apparently be of the order of 0.1 to 0.7 for practical GM from 10 to 30 dB which matches with the result ($d < 0.736$ for stable OPLL operation) given in [9].
Fig. 3 illustrates this by showing the GM as a function of ξ for two different values of d. Also, ξ should be from 0.707 to 1.0 to maintain 10 dB to 30 dB GM, both for modified first- and second-order. Here, the GM of the second-order increases gradually, but for the modified first-order decreases slowly as ξ increases.

In Fig. 4, PM is plotted against d of modified first- and second-order OPLL for two different values of ξ. For modified first- and second-order, d should be within 0.1-0.3 and 0.1-0.7, respectively, to achieve PM from 40^0 to 60^0 which justifies [9].
Fig. 5 shows PM with two different loop filter configurations versus ξ using two different values of d. The value of ξ from 0.707 to 1.0 is sufficient to maintain 40^0 to 60^0 PM for both with loop delay. The phase perturbation of second-order loop is much more susceptible to d than the modified first-order, both for lower and higher damping condition.

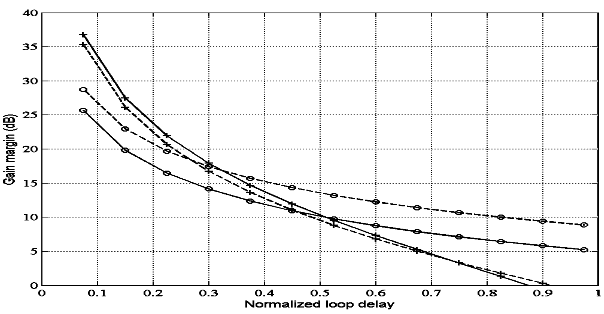

Fig. 2. GM versus d for modified first- and second-order OPLL. The symbols 'o-' and 'o--' are for modified first-order loop with $\xi = 0.707, 1.0$, respectively. The symbols '+-' and '+--' are for second-order loop with $\xi = 0.707, 1.0$, respectively.

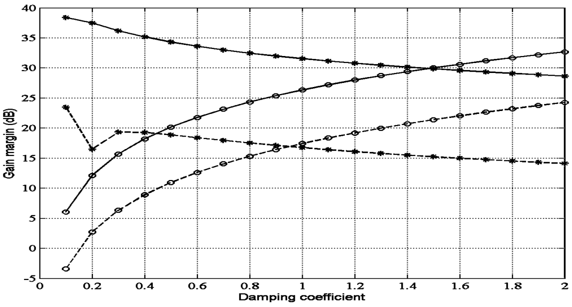

Fig. 3. GM versus ξ for modified first- and second-order OPLL. The symbols 'o-' and 'o--' are for modified first-order loop with $d = 0.1, 0.3$, respectively. The symbols '*-' and '*--' are for second-order loop with $d = 0.1, 0.3$ respectively.

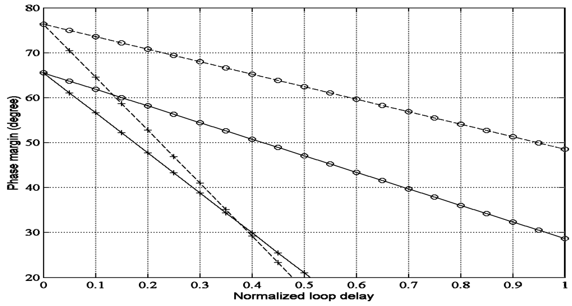

Fig. 4. PM versus d for modified first- and second-order OPLL. The symbols 'o-' and 'o--' are for modified first-order loop with $\xi = 0.707, 1.0$, respectively. The symbols '+-' and '+--' are for second-order loop with $\xi = 0.707, 1.0$, respectively.

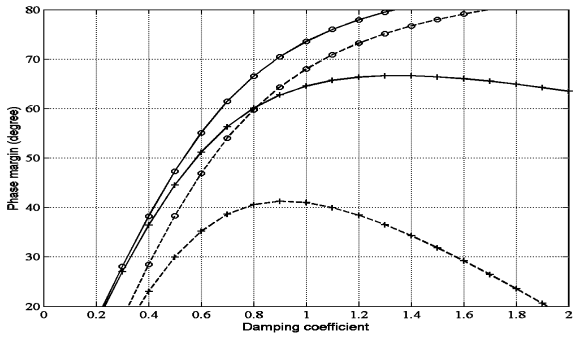

Fig. 5. PM versus ξ for modified first- and second-order OPLL. The symbols 'o-' and 'o--' are for modified first-order loop with $d = 0.1, 0.3$, respectively. The symbols '+-' and '+--' are for second order loop with, $d = 0.1, 0.3$ respectively.

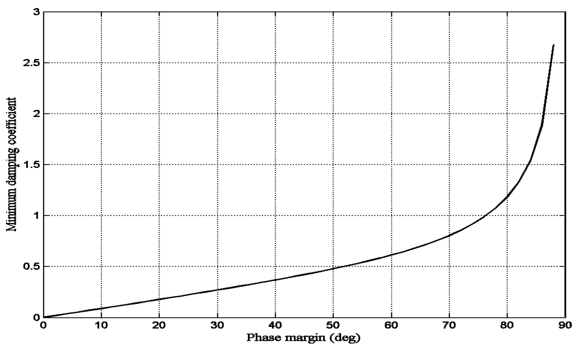

Fig. 6. $\xi_{\min}$ versus PM.

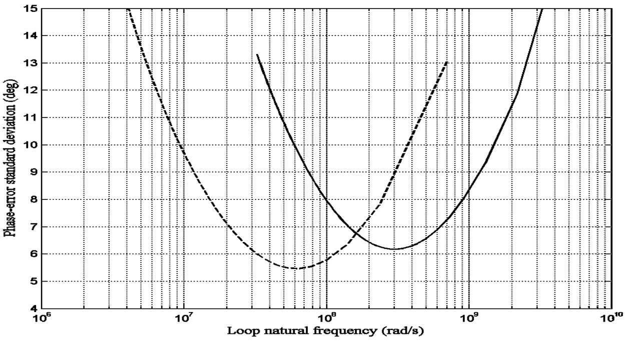

Fig. 7. σ_E versus ω_n for modified first- and second-order OPLL in presence of loop delay. The symbols '--' and '-' are for second- and modified first-order loop, respectively.

Fig. 6 depicts that the minimum damping factor $\xi_{\min}$ is required for a given PM between 0^0 to 90^0. For small PM, $\xi_{\min}$ increases linearly with PM and consistent with ξ i.e. needed for a prescribed PM as in Fig. 4. At higher PM, $\xi_{\min}$ increases more rapidly, and ξ should be large for PM close to 90^0.

Fig. 7, calculated from (20), shows the phase-error standard deviation σ_E for a modified loop as a function of loop-natural frequency ω_n. A summed laser line-width of 5 MHz, 10 dB GM and 45^0 PM are used for each point. P_R is set to -53.0 dBm and $R = 0.94$ A/W. For low ω_n, σ_E is large and is dominated by the laser phase noise. For large ω_n, σ_E is dominated by the shot noise. The PESD reaches a minimum at a particular value of ω_n for both loops. This optimum ω_n for minimum σ_E of a second-order loop (~ 60 Mrad/s) is smaller than that for a modified first-order loop (~ 300 Mrad/s) indicating that second-order loops are more sensitive to loop delay time than modified first-order loop.

IV. CONCLUSION

The loop delay deteriorates the relative stability both for higher and lower damping conditions of first- and second-order OPLL. Expressions of the GM and PM have been derived for both the cases in terms of the damping coefficient and normalized loop delay. The gain perturbations for the second-order OPLL is much less susceptible to damping coefficient than the modified first-order OPLL, both for low and high values of normalized loop delay. Due to the presence of repeated poles at the origin, the second-order OPLL is much more susceptible to phase perturbation than the modified first-order loop with loop propagation delay. The PM of modified first- and second-order OPLL is controlled by the damping co-efficient in absence of loop delay. But, the modified first-order loop can present better performance in terms of PESD than the second-order loop for large loop natural frequencies.

REFERENCES

[1] A. Chiuchiarelli, M. J. Fice, E. Ciaramella and A. J. Seeds, "Effective homodyne optical phase locking to PSK signal by means of 8b10b line coding," Optics Express, pp. 1707-1712, 2011.
[2] M. J. Fice., A.Chiuchiarelli, E. Ciaramella and A. J. Seeds., "Homodyne Coherent Optical Receiver Using an Optical Injection Phase-Lock Loop, Journal of Lightwave Technology," pp. 1152-1164, 2011.
[3] F. Herzog, K. Kudielka, D. Erni and W. Bachtold, "Optical phase locking by local oscillator phase dithering," J. of Quantum Electron., vol.40,pp.973-985,2006.K. Elissa, "Title of paper if known," unpublished.
[4] M. L. Stevens, D. O. Caplan, B. S. Robinson, D. M. Boroson and A.L. Kachelmyer, "Optical homodyne PSK demonstration of 1.5 photons per bit at 156 Mbps with rate-1/2 turbo coding," Opt. Exp., vol. 16, 2008.
[5] A. Banerjee, and B. N. Biswas, "BPSK homodyne receivers based on modified balanced optical phase-locked loop," Optik-Int. J. for Light and Electron Optics, Vol. 124, No. 11, pp. 994-997 (2013).
[6] A. Banerjee, N. R. Das, and B. N. Biswas, "Optical microwave generation using modified sideband injection synchronization with wide line-width lasers for broadband communications," Optik-Int. J. for Light and Electron Optics, vol. 125, No.1, pp. 308-313 (2014).
[7] J. W. M. Bergmans, "Effect of loop delay on phase margin of first-order and second-order control loops," IEEE Trans. Circuits Syst. II, vol. 52, pp.621-625,2005.
[8] A. Banerjee and B. N. Biswas, "Stability analysis of a modified PSK homodyne optical receiver," Int. J. of Electronics and Communication Engg. and Technology, pp. 32-40, 2012.
[9] M. A.Grant, W. C. Michie, and M. J. Fletcher, "The performance of optical phase-locked loops in the presence of nonnegligible loop propagation delay," J. Lightw.Technol., vol. LT-5, pp. 592-597, 1987.
[10] B. C. Kuo, Automatic Control System, 7th ed. India: Prentice Hall, 1997.