4^{th} URSI AT-RASC, Gran Canaria, 19 – 24 May 2024

Impedance matching a cylindrical monopole at the centre of a circular ground plane

Christopher G. Hynes* and Rodney G. Vaughan
School of Engineering Science, Simon Fraser University
Burnaby, BC V5A 1S6, Canada. E-mail: c_h@sfu.ca;rodney_vaughan@sfu.ca

Abstract

The input impedance and impedance matching of a cylindrical monopole at the center of a circular ground plane are summarized. The monopole length, radius, and ground plane radius are varied between 0.23λ to 0.26λ, $10^{-5}\lambda$ to $5\times10^{-3}\lambda$, and 0.2λ to 2.0λ, respectively, where λ is the wavelength. Using numerical results from the Finite Element Method, previous theoretical impedance results for an infinitesimally thin element are shown to poorly model wires of practical thicknesses over mobile communication frequencies. To obtain good antenna impedance matching, i.e., $S_{11} \leq -10$ dB, for any ground plane radius greater than $\lambda/2$ and any practical monopole radius, a monopole length of 0.24λ should be used.

1 Introduction

A monopole over a finite ground plane is one of the simplest but practical antennas. This stems from numerous reasons, including its simple construction, facilitation of a direct connection to a coaxial feed, ability to provide good impedance matching to the 50 or 75 Ω standards without matching components, and mitigation of performance effects from the feed cable via the ground plane. Although rectangular ground planes are easier to fabricate, a circular ground plane can support an azimuthally symmetric radiation pattern. It is well known that a monopole's impedance and bandwidth strongly depend on the wire thickness, length, and size of the ground plane.

A thin wire monopole over a circular ground plane has been well studied since it is tractable for theoretical analysis. It has traditionally been analyzed by assuming the element is infinitesimally thin so that its current can be assumed as sinusoidal. With the element's current known, various theoretical techniques can be used to determine the ground plane's current distribution. These methods include the integral equation method [1] when the ground plane is small compared to wavelength, the method of oblate spheroidal wave functions [2] when the ground plane radius is comparable to a wavelength, and variational methods [3] or theories of diffraction [4] when the ground plane is large in terms of wavelengths. Since the mid-1970s, simulation has become the standard means of analysis. It is essential when the monopole element is relatively thick as its current distribution is no longer sinusoidal – consequently, the current distribution on both the monopole element and the ground plane needs to be determined. Early analysis [5–7] used the Method of Moments (MoM) in combination with the Geometric Theory of Diffraction (GTD) for ground planes with a radius that is large or comparable to a wavelength. The primary goal was focused on validating the MoM through agreement with the measurements [8] and did not investigate the sensitivity versus length and wire diameter. As simulation capabilities improved, the MoM was used alone – initially limited to ground plane radii that were not too large compared to a wavelength [9] and later to a wide range of radii [10]. Hybrid techniques, with or without using the MoM, were still common into the 1990s and 2000s [11–13]. As simulation capabilities improved further, Finite Difference Time Domain (FDTD) simulations were used [14, 15]. However, accurate results for an electrically tiny cylindrical monopole have yet to be published due to the meshing challenges and the large number of mesh cells required.

The most comprehensive investigation of a cylindrical monopole over a circular ground was presented by Weiner [10, 16]. Unfortunately, most impedance results were for impractically thin wires when deployed at mobile radio frequencies, i.e., $10^{-6}\lambda$. Weiner investigated the impedance effects of varying monopole thicknesses, but the results were presented only for resonant lengths – although he does provide resonance lengths for particular wire thicknesses and ground plane radii [16]. Weiner showed that the element's resonant length varied from 0.22 to 0.34λ when the element's radius ranged between $10^{-7}\lambda$ to $10^{-2}\lambda$. The resonant radiation resistance varied from 21 to 65 Ω across the ground plane and element radii.

This paper investigates the impedance characteristics of a cylindrical monopole over a circular ground plane as a function of ground plane radius for different monopole thicknesses and lengths. Given the three degrees of freedom (assuming no matching components) in designing a coaxial-fed cylindrical monopole, the goal is to provide design guidance to achieve good matching. This paper is the first to present results using the Finite Element Method (FEM) to determine the monopole impedance. Although researchers have traditionally focused on modelling the in-

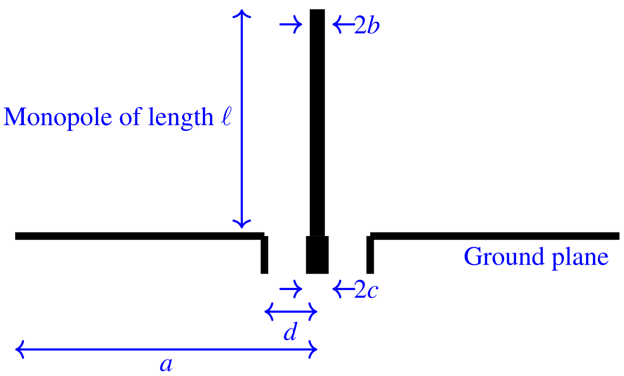


Figure 1. Dimensions of the monopole wire over a circular ground plane fed through a coaxial feed. The coaxial inner conductor and shield radii are $c = 0.006\lambda$ and $d = 0.0139\lambda$, respectively, yielding a 50 Ω feed impedance. The ranges for the ground plane radius a, the monopole length ℓ, and monopole wire radius b are $0.2 \leq a \leq 2.0$, $0.23\lambda \leq \ell \leq 0.26\lambda$, and $10^{-5}\lambda \leq b \leq 5\times10^{-3}\lambda$, respectively.

put impedance or radiation pattern, we provide practical design guidance to achieve a good impedance match to 50 Ω when the monopole length is near $\lambda/4$.

2 Model

It is challenging to simulate a very thin ($b \leq 10^{-5}\lambda$) cylindrical monopole over a circular ground plane using FEM or FDTD due to the large dynamic range of the mesh cell sizes and large number of mesh cells. However, computer capabilities have reached the point where using FEM is a practical solver. For this antenna, FEM has the advantage over FDTD in that it uses tetrahedral mesh cells that conform well to curved surfaces. It has the advantage over MoM by using an adaptive mesh convergence algorithm – allowing for more confidence in the simulation results.

The antennas are simulated using CST Microwave Studio's FEM solver [17]. A diagram of the model is shown in Fig. 1. Based on the electrical sizes of typical wire at mobile communication frequencies, see Table 1, the monopole radius a is varied from $10^{-5}\lambda$ to $5\times10^{-3}\lambda$. The monopole length ℓ ranges from 0.23λ to 0.26λ. The radius of the ground plane a ranges from 0.2λ to 2.0λ. The antenna is fed via a coaxial waveguide port with a line impedance of 50 Ω. All metals are modelled as perfect electrical conductors. The FEM mesh has a maximum mesh segment length of $\lambda/50$ on the model, with mesh refinement on the monopole element and the edges of the ground plane. The FEM mesh convergence criterion is a S-parameter change less than 0.01 for two consecutive mesh refinements.

3 Results and Discussion

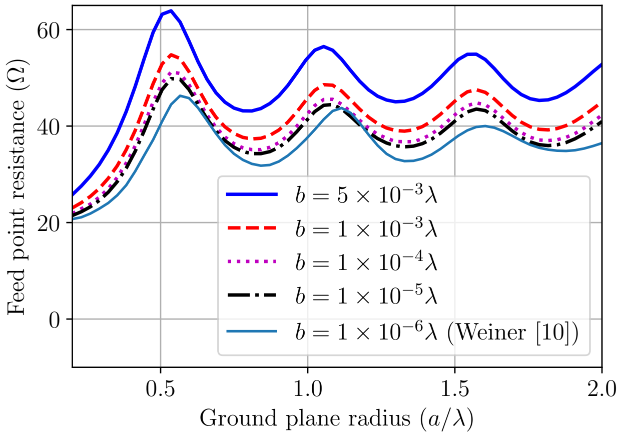
Fig. 2 shows the impedance and matching of a $\lambda/4$ monopole as a function of ground plane radius for different wire thicknesses. The well-know oscillatory variation is seen as the ground plane radius increases. For reference, the impedance results from Weiner [10] are included. They differ slightly from the FEM results because Weiner forced a sinusoidal current on the element. Theoretically derived impedance of monopoles near $\ell = \lambda/4$ assume an infinitesimally thin wire, requiring practical wire radii less than $10^{-6}\lambda$ to be accurate [10]. From Table 1 and Fig. 2, practical wire radii are significantly larger and can not be used to guide the design of a well-matched antenna. Fig. 3 shows the results when the monopole has a length of $\ell = 0.24\lambda$. In contrast to when $\ell = 0.25\lambda$, the reactance versus ground plane radii for $\ell = 0.24\lambda$ varies significant for different wire radii but overall the impedance is better matched to 50 Ω.

Fig. 4 shows a contour plot of the worst-case matching when the ground plane radii are larger than $\lambda/2$. From Fig. 4, good matching, i.e., $S_{11} \leq -10$ dB, can be achieved for all the simulated wire radii whenever the ground plane radius exceeds $\lambda/2$ and the monopole length is 0.24λ. For any other monopole length, there exists a wire and ground plane radius combination that results in an impedance matching $S_{11} \geq -10$ dB.

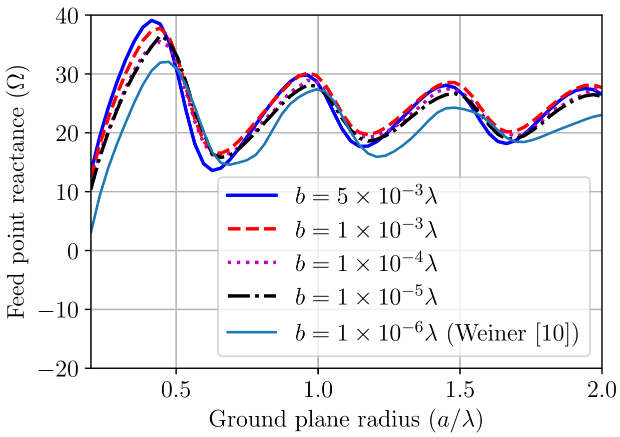
Each of the three design degrees of freedom (wire length, radius, and ground plane radius) strongly impacts the impedance matching. As the ground plane radius increases, the impedance shows the well-known damped oscillatory behaviour that converges to the impedance with an infinite ground plane. As the element length decreases shorter than the resonance near $\lambda/4$, the reactance and resistance sharply decrease, causing the matching to degrade to unacceptable levels. Increasing the length from the $\lambda/4$ resonance, towards $\lambda/2$ anti-resonance, increases both the resistance and reactance, also eventually degrading the matching to unacceptable levels. Furthermore, a monopole longer than $\lambda/2$ increases the lobes in the radiation pattern, which is often unacceptable. Although the matching degrades sharply as the ground plane size decreases less than $\lambda/2$, as seen in Fig. 2c and Fig. 3c, in practice, it would be better due to the currents on the feed cable. However, this is usually undesirable because it can cause the pattern to be erratic, and efforts are often made to suppress the cable currents.

Table 1. The electrical sizes of typical American Wire Gauges (AWG) at mobile communication frequencies.

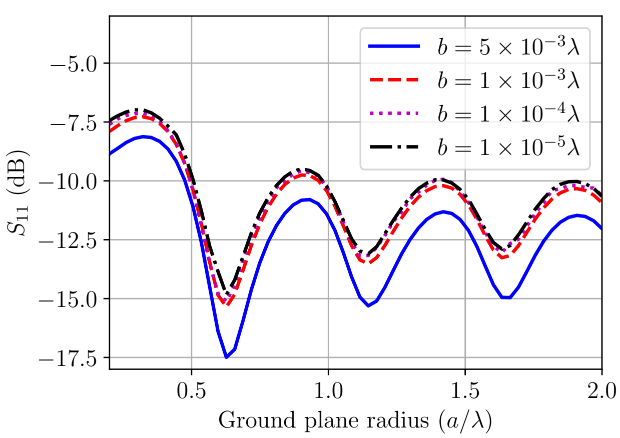
AWG	radius (mm)	wire radius/λ		
		100 MHz	1 GHz	5 GHz
10	1.294	4.3×10^{-4}	4.3×10^{-3}	2.2×10^{-2}
15	0.725	2.4×10^{-4}	2.4×10^{-3}	1.2×10^{-2}
20	0.406	1.4×10^{-4}	1.4×10^{-3}	6.8×10^{-3}
25	0.228	7.6×10^{-5}	7.6×10^{-4}	3.8×10^{-3}
30	0.128	4.3×10^{-5}	4.3×10^{-4}	2.1×10^{-3}
35	0.072	2.4×10^{-5}	2.4×10^{-4}	1.2×10^{-3}



(a)

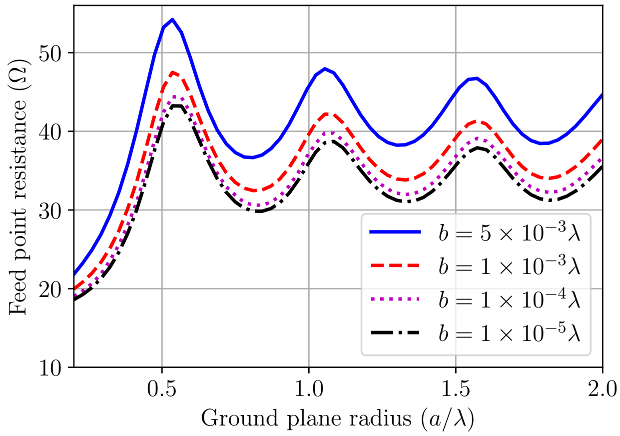


(b)

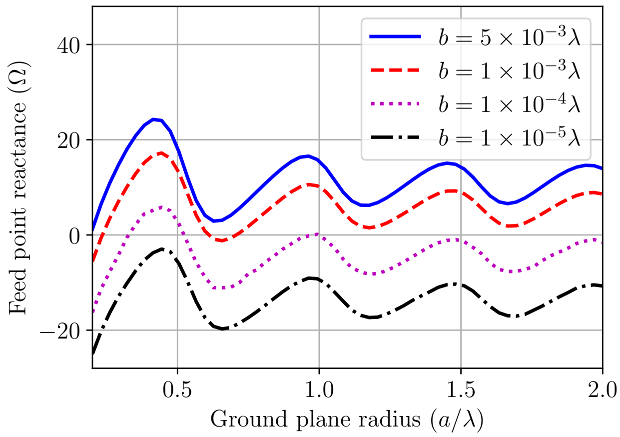


(c)

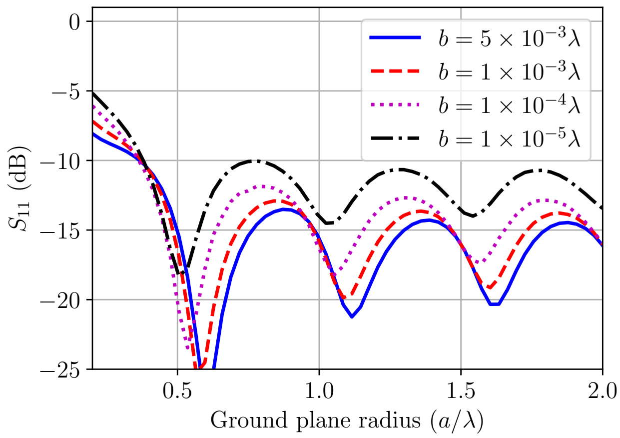
Figure 2. Simulation results for a monopole of length $\ell = 0.25\lambda$ as a function of ground plane radius for different wire radii b: (a) feed point resistance, (b) reactance, and (c) matching.



(a)



(b)



(c)

Figure 3. Simulation results for a monopole of length $\ell = 0.24\lambda$ as a function of ground plane radius for different wire radii b: (a) feed point resistance, (b) reactance, and (c) matching.

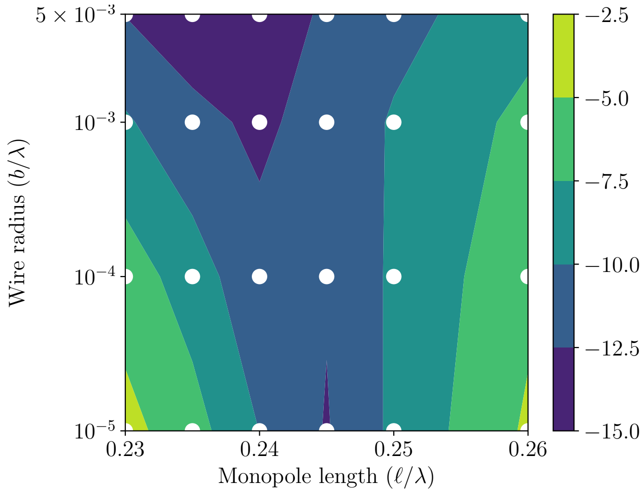


Figure 4. Contour plot of the worst-case matching S_{11} in dB across ground plane radii $a \geq \lambda/2$ as a function of wire radius b/λ and monopole length ℓ/λ. The parameter evaluation sets are shown as white dots.

4 Conclusion

Using numerical results obtained by Finite Element Method simulation, the input impedance and matching of a cylindrical monopole at the center of a circular ground plane are presented. The cylindrical monopole length and radius ranged from 0.23λ to 0.26λ and $10^{-5}\lambda$ to $5 \times 10^{-3}\lambda$, respectively. The ground plane radius ranged from 0.2λ to 2.0λ. It was shown that a well-matched antenna, i.e., $S_{11} \leq -10$ dB, is achieved for practical wire thicknesses when the monopole length is 0.24λ, and the ground plane radius exceeds half-wavelength.

References

[1] J. Bardeen, "The diffraction of a circularly symmetrical electromagnetic wave by a co-axial circular disc of infinite conuctivity," *Phys. Rev.*, vol. 36, pp. 1482–1488, Nov 1930.

[2] A. Leitner and R. D. Spence, "Effect of a Circular Groundplane on Antenna Radiation," *Journal of Applied Physics*, vol. 21, no. 10, pp. 1001–1006, 04 1950.

[3] J. E. Storer, "The Impedance of an Antenna over a Large Circular Screen," *Journal of Applied Physics*, vol. 22, no. 8, pp. 1058–1066, 04 1951.

[4] C. L. Tang, "On the radiation pattern of a base-driven antenna over a circular conducting screen," *Journal of the Society for Industrial and Applied Mathematics*, vol. 10, no. 4, pp. 695–708, 1962.

[5] G. Thiele and T. Newhouse, "A hybrid technique for combining moment methods with the geometrical theory of diffraction," *IEEE Transactions on Antennas and Propagation*, vol. 23, no. 1, pp. 62–69, 1975.

[6] K. Awadalla and T. Maclean, "Input impedance of a monopole antenna at the center of a finite ground plane," *IEEE Transactions on Antennas and Propagation*, vol. 26, no. 2, pp. 244–248, 1978.

[7] ——, "Monopole antenna at center of circular ground plane: Input impedance and radiation pattern," *IEEE Transactions on Antennas and Propagation*, vol. 27, no. 2, pp. 151–153, 1979.

[8] A. Meier and W. Summers, "Measured impedance of vertical antennas over finite ground planes," *Proceedings of the IRE*, vol. 37, no. 6, pp. 609–616, 1949.

[9] J. Richmond, "Monopole antenna on circular disk," *IEEE Transactions on Antennas and Propagation*, vol. 32, no. 12, pp. 1282–1287, 1984.

[10] M. Weiner, "Monopole element at the center of a circular ground plane whose radius is small or comparable to a wavelength," *IEEE Transactions on Antennas and Propagation*, vol. 35, no. 5, pp. 488–495, 1987.

[11] G. A. Thiele, "Overview of selected hybrid methods in radiating system analysis," *Proceedings of the IEEE*, vol. 80, no. 1, pp. 66–78, 1992.

[12] Z. Shen and R. MacPhie, "Input impedance of a monopole over a circular ground-plane of finite thickness," in *IEEE International Symposium on Antennas and Propagation USNC/URSI National Radio Science Meeting*, vol. 3, 1998, pp. 1726–1729 vol.3.

[13] K. Kitagata, A. Inagawa, and S. Tokumaru, "The analysis of monopole antenna centered on circular ground plane using boundary diffraction wave," *Electronics and Communications in Japan (Part I: Communications)*, vol. 88, no. 4, pp. 54–63, 2005.

[14] Z. Živković, D. Senić, C. Bodendorf, J. Skrzypczynski, and A. Šarolić, "Radiation pattern and impedance of a quarter wavelength monopole antenna above a finite ground plane," in *SoftCOM 2012, 20th International Conference on Software, Telecommunications and Computer Networks*, 2012, pp. 1–5.

[15] A. Araghi and R. G. Vaughan, "The effect of a finite circular groundplane on monopole impedance," in *2023 URSI International Symposium on Electromagnetic Theory (EMTS)*, 2023, pp. 1–4.

[16] M. M. Weiner, *Monopole Antennas*. New York: Marcel Dekker, 2003.

[17] Dassault Systèmes, "CST Microwave Studio," 3DS.com https://www.3ds.com/products/simulia/cst-studio-suite (accessed February 1, 2024).