# A Geometry-based Hough Transform Estimation for Radial Velocity of Moving Target Using Single Antenna SAR

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## Abstract

Accurate and real-time radial velocity estimation is a key challenge for the SAR-GMTI system using a single channel. To reduce the heavy computation load *O(MN)* of the conventional Hough transform (HT) estimation (CHTE), an efficient HT estimation (EHTE) is proposed by exploiting the geometry information. The geometry relationship among the range walk angle and the two HT angles is modeled by utilizing the lengths of the HT ranges. Independent of the searching operation, the proposed EHTE not only reduce the computation complexity from *O(MN)* to *O(2N)* but also break through the estimation accuracy bound of CHTE due to the searching step size. Simulation is given to demonstrate the effectiveness of the EHTE method.

## 1. Introduction

Synthetic aperture radar (SAR) – ground moving targets indication (GMTI) has been widely used in civil and military applications [1]. Working at any time with any weather, SAR-GMTI can realize imaging and localization of moving targets in the SAR image effectively and conveniently. However, the procedure of SAR-GMTI is mainly disturbed by the radial velocity of moving target [2]: the range walk due to the radial velocity is not beneficial for moving targets imaging; and the azimuth offset caused by the radial velocity should be measured to relocate the moving targets in the SAR image. So it is important to estimate the radial velocity of moving target for moving targets imaging and localization.

Since the radial velocity brings range walk to the range compression results, the skewed range walk angle can be used to estimate the radial velocity [2]. By searching all possible angles and ranges, the conventional Hough transform (HT) can obtain the best matched range walk angle, and then the radial velocity of moving target can be obtained [3]. However, the 2-D searching operation lead to not only heavy computation load but also the limited estimation accuracy due to the searching step size, which is not recommended in practice. Focusing on accurate and real-time radial velocity estimation by the simple equipment and realization, we propose an efficient HT by exploiting the geometry information. The efficient HT estimation can realize accurate radial velocity estimation with low computation complexity.

## 2. Signal model

Single antenna is adopted to transmit and receive the linear frequency modulated (LFM) signal, and the bandwidth and carrier frequency of the LFM signal is set as $B_r$ and $f_c$. In the illuminated scene of the SAR-GMTI system, a ground moving target travels with constant radial velocity $v_r$ in the synthetic aperture time $T_a$.

The instantaneous slant range $R(\eta)$ from the moving target to the SAR platform can be expressed approximately by the second order Taylor expansion as

$$R(\eta) \approx R_n + v_r\eta + \left(v^2\eta^2\right)/\left(2R_n\right) \tag{1}$$

where $\eta$ is the slow time along the azimuth direction, $R_n$ is the nearest range from the moving target to the track of SAR platform. After range compression, the signal of moving target can be expressed in range compression domain as

$$s(\tau,\eta) = A\cdot\operatorname{sinc}\left[B_r\left(\tau - 2R(\eta)/c\right)\right]\cdot\operatorname{rect}\left[\eta/T_a\right]\exp\left(-j4\pi f_c R(\eta)/c\right) \tag{2}$$

where $A$ is the amplitude term, $\tau$ is the fast time in range, and $c$ is the speed of light.

According to the combination of (1) and (2), the trajectory of the moving target presents as a skewed line in the range compression domain, i.e. the range walk occurs. Compared with the range walk caused by $v_r$, the range curvature due to $v^2/2R_n$ is so small that can be ignored in the airborne SAR system. Since the range walk is brought by the radial velocity of moving target, the skewed angle of the range walk can be used to estimate the radial velocity. Conveniently, the skewed angle of the range walk is named as range walk angle. The theoretical range walk angle of moving target is set as $\theta$, as shown in Fig.1(a). Then the relationship between $\theta$ and $v_r$ can be obtained by the geometry in the range compression domain

$$v_r = c\cdot\tan\theta\cdot f_{sa}/2f_{sr} \tag{3}$$

where $f_{sa}$ and $f_{sr}$ are the PRF of the system and the sampling frequency in range direction.

Thus, estimating for the radial velocity of moving target is transformed into estimating the range walk angle. Hough transform (HT), which is skilled at estimating the skewed angle in 2-D plane, is usually used to estimate the radial velocity of moving target. Under the fixed angle $\varphi$ and range $\rho$, the $\varphi$-HT of a trajectory can be expressed as [4]

$$S_R(\rho,\varphi)=\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\left|s(x,y)\right|\delta(\rho-x\cos\varphi-y\sin\varphi)\,dxdy \tag{4}$$

where $\left|s(x,y)\right|$ is the amplitude of the trajectory at the coordinates of $(x,y)$, and $\delta(\cdot)$ is the pulse function.

By analyzing (4), we find the $\varphi$-HT can be explained as the energy accumulation or projection of the trajectory on the angle of $\varphi$, as shown in Fig.1(a). Based on this explanation of the HT, the Hough transform can be used to estimate the range walk angle or the radial velocity of moving target.

In the conventional Hough transform estimation (CHTE) method, the best matched range walk angle $\hat{\theta}_m$ can be obtained by searching the maximum energy of all the HTs corresponding to all possible angles and ranges, as shown in Fig.1(b). And then the radial velocity of moving target can be calculated by the searched $\hat{\theta}_m$ and (3). However, the angles and ranges (2-D) searching operation brings huge computation complexity of *O(MN)*, where *M* and *N* are the searched number of angles and ranges respectively. Moreover, the estimation accuracy of CHTE is mainly restricted by the searching step size which also determines the computation complexity. So the tradeoff between the estimation accuracy and the computation complexity should be considered when utilizing the CHTE method. The heavy computation load and the tradeoff block the development of the CHTE method in practice.

## 3. Efficient radial velocity estimation methods

In this part, an efficient HT estimation (EHTE) is proposed to solve the problems of the CHTE method. Instead of searching all angles, only two angles are used to calculate the range walk angle and the radial velocity of moving target.

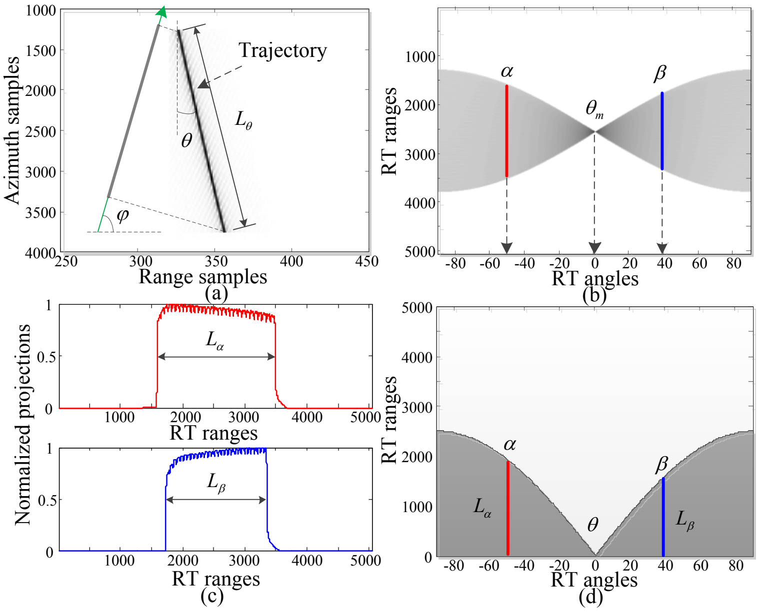

Fig.1. Sketch of the EHTE. (a) Range compression results of moving target; (b) CHTE method; (c) Normalized projections of $\alpha$-HT and $\beta$-HT; (d) Geometry relationship.

In the CHTE method, the HTs of all angles are shown as the shadow in Fig.1(b),where the edges of the shadow present as two smooth curvatures. Naturally, we are wondering whether there exist some function about the edges or not. If the function of the curvatures can be found, then the range walk angle can be obtained by the optimal value of the function. In order to reduce the two curvatures to one curvature, the HTs are accumulated along the direction of HT ranges, with the results shown as Fig.1(d). Obviously, the minimum of the curvature's function in Fig.1(d) is corresponding to the range walk angle of moving target. So we aim at obtaining the function $f(\phi)$ of the curvature in Fig.1(d).

On the other hand, since $\varphi$ -HT can be explained as the projection of the trajectory on the angle of $\varphi$ , the function of the curvature must be a sine based function, which can be obtained from the geometry relationship in Fig.1(a). We set the function $f(\phi)$ as

$$f(\phi) = a\sin\left(k\phi\right) = b\sin\left(\phi\right) \tag{5}$$

where $a$ and $k$ are constant. Considering the minimum value of $f(\phi)$ for $\theta$ as shown in Fig.1 (d), the function should be modified to

$$f(\phi) = b\sin\left(\phi - \theta\right) \tag{6}$$

Two angles are used to cancel the constant term in (6). As shown in Fig.1(b), two angles ( $\alpha$ and $\beta$ ) are used to get the corresponding HTs ( $\alpha$ -HT and $\beta$ -HT) of the trajectory of moving target. And the normalized projections of $\alpha$ -HT and $\beta$ -HT are shown in Fig.1(c), where the lengths of the normalized projections (LNP) can be measured as $L_\alpha$ and $L_\beta$ . The LNPs can be explained as the length between the nearest and farthest HT ranges of the trajectory. As shown in Fig.1(d), the absolute value of $f(\alpha)$ and $f(\beta)$ should be $L_\alpha$ and $L_\beta$ . So the expression of (6) can be applied for $\alpha$ and $\beta$ as

$$f(\alpha) = b\sin\left(\theta - \alpha\right) = L_\alpha \tag{7}$$

$$f(\beta) = b\sin\left(\beta - \theta\right) = L_\beta \tag{8}$$

where $\theta - \alpha > 0$ and $\beta - \theta > 0$ can be realized because of the limited range walk angle of ground moving target.

After division between (7) and (8), we can obtain the following geometry relationship

$$\frac{f(\alpha)}{f(\beta)} = \frac{\sin\left(\theta - \alpha\right)}{\sin\left(\beta - \theta\right)} = \frac{L_\alpha}{L_\beta} \tag{9}$$

Then the range walk angle of moving target can be calculated by (9) as

$$\tan\hat{\theta} = \left(L_\beta \sin\alpha + L_\alpha \sin\beta\right) / \left(L_\beta \cos\alpha + L_\alpha \cos\beta\right) \tag{10}$$

And the radial velocity of moving target can be obtained by (10) and (3) as

$$\hat{v}_r = cf_{sa}\left(L_\beta \sin\alpha + L_\alpha \sin\beta\right) / 2f_{sr}\left(L_\beta \cos\alpha + L_\alpha \cos\beta\right) \tag{11}$$

Only two angles are used to get the HTs of the trajectory of moving target, and the geometry relationship between the range walk angle and the two HT angles is modeled by exploiting the geometry information. The geometry relationship can be used to calculate rather than search the range walk angle of moving target and the radial velocity of moving target. Thus, the computation complexity of the proposed EHTE method is reduced from *O(MN)* to *O(2N)*. Furthermore, independent of the searching operation, the estimation accuracy of EHTE is not limited by the searching step size. Optimistically, the EHTE would possess much higher estimation accuracy than that of CHTE, which will be demonstrated by the following experimental results.

## 4. Experimental results and analysis

Simulation is presented to demonstrate the validity of the proposed EHTE method. Since the computation complexity of EHTE has been mentioned above, the estimation accuracy of EHTE is analyzed in this section.

Under different output signal to noise ratio (SNR) in the range compression domain, the estimation accuracy of CHTE and EHTE are compared, with the estimation results of the radial velocity shown as Fig.2. Obviously, the stable estimation is achieved by CHTE and EHTE when SNR>0dB and SNR>22dB respectively. But when SNR>18dB, the relative error of the estimated radial velocity by EHTE is lower than that by CHTE. These appearances illustrates that the EHTE possesses much higher estimation accuracy than CHTE in the high SNR case. This is because the geometry based EHTE may be disturbed in low noise circumstance. On the other hand, since EHTE is independent of the searching operation, EHTE would behave obvious advantage in high estimation accuracy when the SNR is improved.

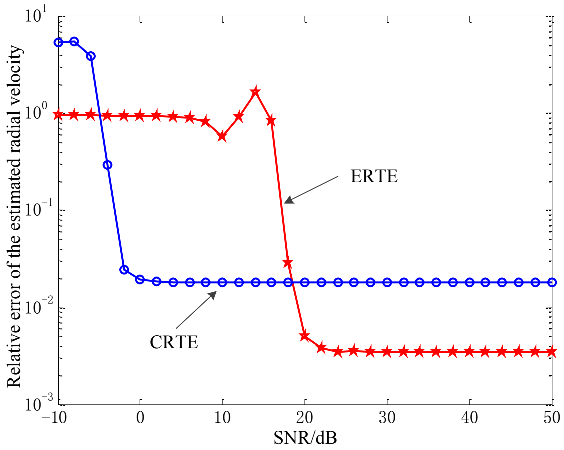

Fig.2 Estimation accuracy comparison results in noise background

## 5. Conclusions

An efficient HT is proposed to estimate the radial velocity of moving target in the SAR-GMTI system. By exploiting the geometry relationship, we effectively reduce the computation complexity from *O(MN)* to *O(2N)*. Independent of the searching operation, the proposed EHTE method can calculate rather than search the radial velocity. Furthermore, the estimation accuracy of EHTE is higher than that of the conventional HT estimation method. Sensitive to noise, the proposed EHTE is good at accurate and real-time estimation in the high SNR circumstance.

As a primary research, this paper presents the advantages of utilizing the geometry information in application. And the geometry information exploiting should be encouraged in the field of signal processing.

## 6. Acknowledgment

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## 7.References

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