Transmission-Line Model for a Non-linear and Dispersive Parity-Time (PT) Symmetric Structure

Abstract

1. Introduction

2. Parity-Time (PT) Symmetric Structure

3. Transmission-Line Model for Nonlinear and Dispersive Gain/loss Medium
\[ E \leftrightarrow \frac{V}{\Delta \ell} ; H \leftrightarrow -i \frac{\Delta \ell}{\Delta \ell \eta_0} ; \sigma_0 \leftrightarrow \frac{g_e}{\Delta \ell \eta_0} ; p_x = -\frac{p_y \Delta \ell}{\varepsilon_0}, \]

where the constants are given by

\[ \frac{\Delta \ell \Delta \ell}{\Delta \ell} = \Delta \ell \eta_0 \]

Due to [\ref{eq:1}], a non linear material can be described by

\[ S(l) = \frac{1}{1 + (l/l_{sat})}. \]

The Duffing model can be expressed as

\[ \frac{\partial^2 P_D}{\partial T^2} + K_{D1} \frac{\partial P_D}{\partial T} + K_{D2} P_D f_0(p_{cy}) = K_{D3} V_y. \]

Equation (8) is now ready to be implemented within the TLM algorithm.

### 3.2 Duffing Model of Nonlinear Materials

\[ P_e = \chi_0 |E| + \chi_1^{(2)} |E|^2 + \chi_2^{(3)} |E|^3. \]

### 3.3 Implementation of the Digital Filter

\[ K_{e2} V_y + 2p_{dy} = 2V_y' + z^{-1}S_{ey}. \]

\[ K_{e1} = -(2 + g_e - 2\Delta \chi_e) ; K_{e2} = 2 + g_e + 2\Delta \chi_e \]

\[ S_{ey} = 2V_y' + K_{e1} V_y + S_{ec} + 2p_{dy} ; S_{ec} = \hat{g}(z) V_y. \]

\[ p_{dy} K_{d4} + p_B K_{d2} f_0(p_{cy}) + z^{-1}S_{d1} = K_{d3} V_y. \]

\[ K_{d4} = 4 + 2K_{d1} \]

\[ S_{d1} = p_D (-8 + 2K_{d2} f_0) - 2K_{d3} V_y + z^{-1}S_{d2} \]

\[ S_{d2} = p_D (4 - 2K_{d1} + K_{d2} f_0) - K_{d3} V_y. \]

The Duffing model is expressed as

\[ \frac{\Delta \ell \Delta \ell}{\Delta \ell} = \Delta \ell \eta_0 \]

The parameters and their implementation are described in the following subsection, corresponding material parameters are related to the physical electromagnetism. 

In this paper, describable dispersive and saturable gain/loss model is assumed

\[ K_e - \sigma_0 \Delta \ell \rightarrow \frac{g_e}{\Delta \ell \eta_0}, \]

where, arising from the circuit equivalences, eq. (4) can be described by

\[ K_e - \sigma_0 \Delta \ell \rightarrow \frac{g_e}{\Delta \ell \eta_0} \]

\[ (1 + z^{-1})g_e = g_{e0} + z^{-1} \hat{g}(z), \]

\[ g_{e0} = g \left( \frac{K_3}{K_6} \right) \]

\[ \hat{g}(z) = \frac{b_0 + z^{-1}b_1 + z^{-2}b_2}{1 - z^{-1}(-a_1) - z^{-2}(-a_2)} \]

\[ K_4 = \frac{1}{\tau}; K_2 = 1 + (\omega_0 \tau)^2/2 \]

\[ K_3 = 2K_1 \Delta t + K_2^2 \Delta t^2; K_4 = 2(K_1 \Delta t) \]

\[ K_5 = -2K_1 \Delta t + K_2^2 \Delta t^2; K_6 = 4 + 4K_1 \Delta t + K_2 \Delta t^2 \]

\[ K_7 = -8 + 2K_2 \Delta t; K_8 = 6 + 4K_1 \Delta t + 2K_3 \Delta t^2 \]

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4. A Non-linear PT Memory Device

![Figure 1](image1.png)

**Figure 1.** Schematic illustration of an input signal used in the detection process. The transmission from the right port is shown in Fig. 2(b) with a high similarity to the PT grating forms a hysteresis profile with the transmission from the right port.

**Table 1.** Parameters related to the 1D gain/loss material parameter is given in Fig. 2 as the function of the input intensity, for different values of frequency-dependent material parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$n_{\text{lo}}$</th>
<th>$n_{\text{hi}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\chi_{\text{eo}}$</td>
<td>$\Delta \chi_{\text{eo}}$</td>
<td>$\delta \text{rad/ps}$</td>
</tr>
<tr>
<td>$200$ cm$^{-1}$</td>
<td>$336.85$</td>
<td>$3.626$</td>
</tr>
</tbody>
</table>

The equation for the dispersive gain/loss material parameter is considered as a non-linear PT Memory Device. For different values of frequency-dependent material parameters, the equation is related to the imaginary part of the refractive index by $\alpha_l = \frac{\omega_p}{c_0} \eta''(\omega_p)$.

For the sake of simplicity, the transmittance of the PT grating is shown in Fig. 2 as the function of the input intensity, for different values of frequency-dependent material parameters.

**Figure 2.** Transmittance left $T_L$ and right $T_R$ as a function of intensity $I$ for different values of frequency-dependent material parameters. The switching happens at $T_L < T_R$, which violates the Lorentz reciprocity condition. The inequality is one of the main features of the PT memory property; a repeated sequence of write, read, write, and reset operation gives a high similarity to the PT grating forms a hysteresis profile with the transmission from the right port.

1. **Read Operation:**
   - After the write operation (set state to “0”), the following read operation gives a high similarity to the PT grating forms a hysteresis profile with the transmission from the right port.
transmittance for state “1”, the reset operation brings to “0” and so forth.

5. Summary

A versatile all-optical bistable operation of the implemented within the PT gallery mode structure described in this paper can be used to model any arbitrary nonlinear parity processing device using a Bragg grating as in [2,3] or topological invariant as in [3,6].

It is noted that the extended TLM method can be exploited as in [1,2] or topological invariant as in [3,6].

The transmitted electric field for left incidence and right incidence are presented in Figure 3.

Figure 3. a) Incident signal and transmitted signal for left incident.

6. References

S. Phang, A. Vukovic, H. Susanto, S. C. Creagh, P. N. Meisinger, A. A. Benson, and T. M. Benson, “Demonstration of nonlinear parity memory device. (a) the input electric field and (b) the transmittance for state “1”, the reset operation brings to “0” and so forth.

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