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A First Principles, Multipole-Based Cable Braid Electromagnetic Penetration Model

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Abstract

We report in this paper a first principles, multipole-based cable braid electromagnetic penetration model. We apply this formulation to the case of a one-dimensional array of wires, which can be modeled analytically via a multipole-conformal mapping expansion for the wire charges and extension by means of Laplace solutions in bipolar coordinates. We analyze both electric and magnetic penetrations and compare results from the first principles cable braid electromagnetic penetration model to those obtained using the multipole-conformal mapping expansion method. We find results in very good agreement when using up to the octopole moment (for the first principles model), covering a dynamic range of radius-to-half-spacing ratio up to 0.6. These results give us the confidence that our first principles model works within the geometric characteristics of many commercial cables.

1. Introduction

This paper discusses a first principles, multipole-based cable braid electromagnetic penetration model [1-4]. Canonic parameters are usually used to model a shielded cable [1, 2, 5-9]: some model the shield properties related to the braid weave characteristics and material, namely the per-unit length transfer impedance Z_T (proportional to the transfer inductance L_T and resistance R_T) and transfer admittance Y_T (proportional to the transfer capacitance C_T). Other important parameters are the per-unit length (series) self-impedance Z_c and (shunt) self-admittance Y_c , which are formed by the inner conductor and the shield.

Our goal here is to apply for the first time a first principles model [1-4] that delivers results based on the geometrical parameters of the cable in question. In this paper, we will confirm our first principles model against a canonical structure, namely a one-dimensional array of wires, or wire grid, which can be modeled analytically via a multipole-conformal mapping expansion for the wire charges and extension by means of Laplace solutions in bipolar coordinates [10, 11].

2. One-Dimensional Array of Wires: Electric Penetration

The problem of field leakage through an array of cylinders is the basic canonical periodic shield [12]. When the cylinder radius is small compared with the spacing, simple approximate solutions to this problem can be found [7, 13, 14].

We consider here the effects of line multipole additions to the simple filament approximation in representing the elements of a one-dimensional wire grid array shown in Fig. 1. We look at the limit of small wire radius a (as well as general ratios of wire radius to wire half spacing w) to determine which of the existing approximations to the wire array transfer elastance (the inverse of capacitance [14]) is most accurate. We also construct a simple and accurate uniform approximation using the simple conformal mapping filament approximation along with the decay factor from the solution to Laplace's equation in bipolar coordinates [11, 15].

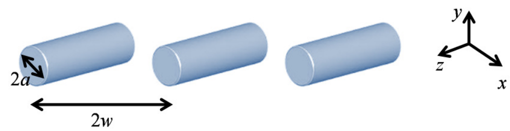

Figure 1. A one-dimensional array of wires with wire radius a and period $2w$.

The array of wires in Fig. 1 is periodic in x (one wire is positioned at $x = 0$) and all wires are parallel with the z axis, each having a line charge density q with wire spacing $2w$ and wire radius a. The transfer elastance of the grid can be defined by $S_c = \phi_c/(wq) = \phi_c/(2w\varepsilon_0 E_0 w)$ [11], where ϕ_c is the difference of the electric potential at the point $y \to +\infty$ and a point on the wire, say at $x=0$, $y=a$ or at $x=a$, $y=0$. The simple small radius a approximation [11], denoted as thin wire leads to

$$S_{c,tw}(2\pi\varepsilon_0 w) \approx \ln\left(\frac{w}{\pi a}\right). \qquad (1)$$

When the cylinders are closely spaced, the attenuation resulting from the region between cylinders is difficult to represent by means of the multipole expansion. To treat this problem, a "smoothed" conformal transformation was used [14], leading to

$$S_{c,sc}(2\pi\varepsilon_0 w) = \ln\left[\csc\left\{\frac{\pi a}{2w}(1+\lambda)\right\}\right]. \qquad (2)$$

The "smoothed" conductor solution is a good approximation to the cylinder only when the conductors are not in close proximity. We construct an accurate approximation to the elastance using a solution to Laplace's equation in bipolar coordinates [15] which holds for all ratios of radius to spacing and is given by [11]

$$S_{c,bs}(2\pi\varepsilon_0 w) \approx -\ln\left(1-e^{-\pi a/w}\right) \times \exp\left[-2\pi \frac{\arctan\left(c/\sqrt{w^2/a^2-1}\right)}{\ln\left(w/a+\sqrt{w^2/a^2-1}\right)}\right], \qquad (3)$$

with $c = \frac{1}{2}\sqrt{1+\frac{\pi a}{2w}}$.

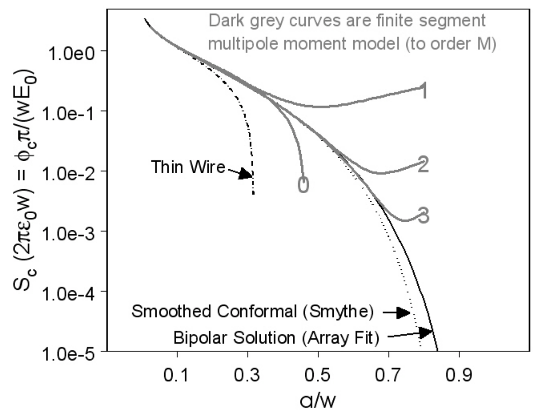

Figure 2. The transfer elastance of a wire grid computed versus the ratio a / w. The black lines pertain to Eqs. (1)-(3). The dark grey lines pertain to the first principles electric multipole penetration model up to order M = 0 (filament), 1 (dipole), 2 (quadrupole), 3 (octopole).

The three transfer elastances in Eqs. (1-3) are reported in Fig. 2. One can see that $S_{c,tw}$ in Eq. (1) is the least accurate; better accuracy is obtained with $S_{c,sc}$ in Eq. (2); finally, the best accuracy is achieved with $S_{c,bs}$ in Eq. (3).

The cable penetration model is based on electric multipoles. We determine ϕ_c/E_0 by solving for the potential surrounding a periodic cell of the structure [1, 2]. The drive potential in the planar problem will be taken as $\phi_{inc} = -E_0 y$ where $y=0$ is at the center of the structure. It is efficient to represent the electric scalar potential by an electric multipole summation to capture the transverse field behavior. The lattice parameters are used to image the potential contribution of an axially varying line charge $q(s)$, discretized as pulses of strength q_n in one periodic cell over the planar structure model. We include a series of line multipole moments in the potential, which for a given position *n*, is written as

$$\phi_{scatt}^{n} = \frac{-1}{4\pi\varepsilon} \sum_{m=0}^{M} \underline{p}^{(0)} \underline{p}^{(1)} \cdots \underline{p}^{(m)} \cdot \nabla_t^m \times \ln\left[\frac{(s - s_n/2) + \sqrt{\rho^2 + (s - s_n/2)^2}}{(s + s_n/2) + \sqrt{\rho^2 + (s + s_n/2)^2}} \right], \tag{4}$$

and the total potential is $\phi_{scatt}^{tot} = \sum_{n=1}^{N} \sum_{j=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} \phi_{scatt}^{n}$. The final matching equation to determine the $2M$ multipole moments on each of *N* segments imposes the constant $\phi_{scatt}^{tot} + \phi_{inc} = V_{n'} = \phi$. Once the potential ϕ is found, with the potential on the braid taken to vanish, $V_{n'} = 0$, we can proceed to find the asymptotic potential constant behaviors of interest. For the shadow side of the structure, we evaluate a total potential far from the braid to find $\phi \to \phi_c$. Normalizing by the drive field E_0 , we find the desired ϕ_c / E_0 .

Using ϕ_c / E_0 and considering up to order M = 0 (filament), 1 (dipole), 2 (quadrupole), 3 (octopole), we plot in Fig. 2 the transfer elastance versus a/w as dark grey curves. One can notice that the agreement with $S_{c,bs}$ is best when using up to the octopole moment, covering a dynamic range of up to $a/w = 0.6$.

3. One-Dimensional Array of Wires: Magnetic Penetration

This is a dual problem to the electric penetration model of an array of wires discussed in Sec. 2 [10]. One can achieve analytical expressions similar to Eqs. (1)-(3) also in this case, and the three transfer inductances are reported in Fig. 3. Again, the best accuracy is achieved with the bipolar solution.

The cable penetration model is based on magnetic multipoles [1, 2]. Considering up to order *M* = 3 (octopole), we plot in Fig. 3 the transfer inductance versus a/w as dark grey curve. One can notice that the agreement with the bipolar solution is best up to $a/w = 0.6$.

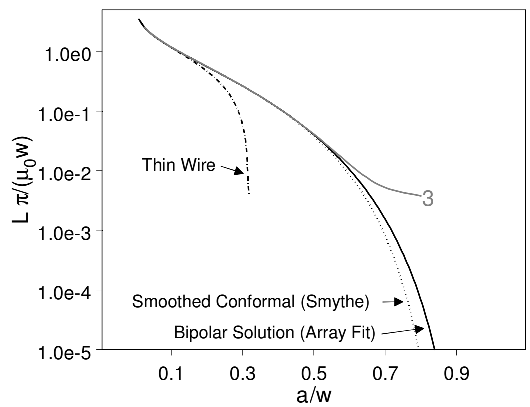

Figure 3. The transfer inductance versus the ratio *a* / *w*. The black lines pertain to dual expressions to Eqs. (1)-(3). The dark grey line pertains to the first principles magnetic multipole penetration model up to order *M* = 3 (octopole).

4. Conclusion

We have reported a first principles, multipole-based cable braid electromagnetic penetration model. We have studied the case of a one-dimensional array of wires for which we report modeling based on a multipole-conformal mapping bipolar solution for the wire elastance. We compared the transfer electric and magnetic parameters from the first principles penetration model to the ones obtained using the analytical method. These results were found in good agreement up to a radius to half spacing ratio of 0.6, within the characteristics of many commercial cables. Our proposed first principles multipole model accounts for the dependence on the actual cable geometry.

5. Acknowledgements

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