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Coupled electromagnetic TE-TE wave propagation in a medium with arbitrary nonlinear saturation

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Abstract

The paper treats a problem of propagation of coupled electromagnetic TE-TE waves in a nonlinear plane waveguide located between two half-spaces with constant permittivities. Nonlinearity in the waveguide is described by a saturated nonlinearity. The physical problem is reduced to a nonlinear two-parameter eigenvalue problem for a system of (nonlinear) ordinary differential equations. Uniqueness of solution to the two-parameter eigenvalue problem is proved. Numerical results are presented.

1 Introduction

For many years electromagnetic polarized wave propagation in nonlinear dielectric waveguides is intensively investigated [1, 2, 3, 4, 5, 6, 7, 8]. The further step was to study the processes of coupled electromagnetic waves propagation in nonlinear waveguide structures [9, 10, 11].

In this paper coupled electromagnetic TE-TE wave propagation problem is investigated. The wave propagates in a nonlinear dielectric waveguide placed between two linear half-spaces. The nonlinearity in the waveguide is described by a monotonically increasing bounded function. The problem is reduced to the multiparameter eigenvalue one w.r.t. two propagation constants. Uniqueness conditions to the two-parameter eigenvalue problem are found.

In addition, we have carried out two numerical experiments for widely used type of nonlinearity with saturation. In each experiment we compared the nonlinear and linear cases. A numerical method for the determination of the coupled propagation constants was presented in [12].

2 Statement of the problem

Let us consider the propagation of an electromagnetic TE-TE wave $\tilde{\mathbf{E}}$, $\tilde{\mathbf{H}}$, where

$$\tilde{\mathbf{E}} = \mathbf{E}_1 e^{-i\omega_1 t} + \mathbf{E}_2 e^{-i\omega_2 t}, \quad \tilde{\mathbf{H}} = \mathbf{H}_1 e^{-i\omega_1 t} + \mathbf{H}_2 e^{-i\omega_2 t},$$

ω_1, ω_2 are the circular frequencies. Complex amplitudes $\mathbf{E}$, $\mathbf{H}$ have the form $\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2$, $\mathbf{H} = \mathbf{H}_1 + \mathbf{H}_2$, where

$$\mathbf{E}_1 = (0, E_{1y}, 0)^\top, \quad \mathbf{H}_1 = (H_{1x}, 0, H_{1z})^\top,$$
$$\mathbf{E}_2 = (0, 0, E_{2z})^\top, \quad \mathbf{H}_2 = (H_{2x}, H_{2y}, 0)^\top,$$

$(\,\cdot\,)^\top$ is the transposition operation, the field components E_{1y}, H_{1x}, H_{1z} do not depend on y and the field components E_{2z}, H_{2x}, H_{2y} do not depend on z.

Waves propagate through a nonlinear homogeneous isotropic nonmagnetic dielectric waveguide Σ, where

$$\Sigma := \left\{(x,y,z) : 0 < x < h, (y,z) \in \mathbb{R}^2\right\},$$

$h > 0$ is a thickness of the waveguide. The permittivity in the entire space has the form $\varepsilon = \varepsilon_0 \tilde{\varepsilon}$, where

$$\tilde{\varepsilon} = \begin{cases} \varepsilon_1, & x > h \\ \varepsilon_2 + \alpha f\left(|\mathbf{E}|^2\right), & 0 \leqslant x \leqslant h, \\ \varepsilon_3, & x < 0, \end{cases}$$

$\varepsilon_0 > 0$ is the permittivity of free space, $\varepsilon_1, \varepsilon_2, \varepsilon_3, \alpha, \beta > 0$ are real constants, f is a monotonically increasing bounded function, $f(0) = 0$, auxiliary condition on the function f will be described in section 3. We assume that $\varepsilon_2 > \max\{\varepsilon_1, \varepsilon_3\}$ and $\min\{\varepsilon_1, \varepsilon_3\} \geqslant \varepsilon_0$.

Maxwell's equations have the form

$$\begin{cases} \operatorname{rot}\left(\mathbf{H}_1 e^{-i\omega_1 t} + \mathbf{H}_2 e^{-i\omega_2 t}\right) = -i\omega_1 \varepsilon \mathbf{E}_1 e^{-i\omega_1 t} - \\ \qquad\qquad -i\omega_2 \varepsilon \mathbf{E}_2 e^{-i\omega_2 t}, \\ \operatorname{rot}\left(\mathbf{E}_1 e^{-i\omega_1 t} + \mathbf{E}_2 e^{-i\omega_2 t}\right) = i\omega_1 \mu_0 \mathbf{H}_1 e^{-i\omega_1 t} + \\ \qquad\qquad + i\omega_2 \mu_0 \mathbf{H}_2 e^{-i\omega_2 t}. \end{cases} \tag{1}$$

Thus the complex amplitudes $\mathbf{E}, \mathbf{H}$ satisfy equations (1) and the radiation condition at infinity, where electromagnetic field exponentially decays as $|x| \to \infty$ in the half-spaces $x > h$ and $x < 0$; in addition, $\mathbf{E}, \mathbf{H}$ satisfy the continuity condition for the tangential field components at the boundaries $x = h$ and $x = 0$.

The components E_{1y}, H_{1x}, H_{1z}, E_{2z}, H_{2x}, H_{2y} have the form

$$E_{1y} = \mathrm{E}_{1y}(x) e^{i\gamma_1 z}, \quad H_{1x} = \mathrm{H}_{1x}(x) e^{i\gamma_1 z}, \quad H_{1z} = \mathrm{H}_{1z}(x) e^{i\gamma_1 z},$$
$$E_{2z} = \mathrm{E}_{2z}(x) e^{i\gamma_2 y}, \quad H_{2x} = \mathrm{H}_{2x}(x) e^{i\gamma_2 y}, \quad H_{2y} = \mathrm{H}_{2y}(x) e^{i\gamma_2 y},$$

where γ_1, γ_2 are unknown real constants, $\mathrm{E}_{1y}, \mathrm{H}_{1x}, \mathrm{H}_{1z}$, $\mathrm{E}_{2z}, \mathrm{H}_{2x}, \mathrm{H}_{2y}$ are unknown functions.

Substituting $\mathbf{E}, \mathbf{H}$ with the above defined components into Maxwell's equations (1), we obtain

$$\begin{cases} \mathrm{E}''_{1y} - \gamma_1^2 \mathrm{E}_{1y} = -k_1^2 \tilde{\varepsilon} \mathrm{E}_{1y}, \\ \mathrm{E}''_{2z} - \gamma_2^2 \mathrm{E}_{2z} = -k_2^2 \tilde{\varepsilon} \mathrm{E}_{2z}, \end{cases} \tag{2}$$

where $k_j^2 = \omega_j^2 \mu_0 \varepsilon_0$ and everywhere integer index $j = 1,2$. The components of the magnetic fields have the form

$$\mathrm{H}_{1x} = -\frac{\gamma_1}{\omega_1 \mu_0}\mathrm{E}_{1y}, \quad \mathrm{H}_{1z} = -\frac{i}{\omega_1 \mu_0}\mathrm{E}'_{1y},$$
$$\mathrm{H}_{2x} = \frac{\gamma_2}{\omega_2 \mu_0}\mathrm{E}_{2z}, \qquad \mathrm{H}_{2y} = \frac{i}{\omega_2 \mu_0}\mathrm{E}'_{2z}.$$

System (2) is linear in the half-spaces $x > h$ and $x < 0$, respectively. Using the radiation condition at infinity, we get its solutions

$$\begin{cases} \mathrm{E}_{1y} = A_1 e^{-\kappa_{11}(x-h)}, \\ \mathrm{E}_{2z} = A_2 e^{-\kappa_{21}(x-h)}, \end{cases} \qquad x > h, \qquad (3)$$

$$\begin{cases} \mathrm{E}_{1y} = B_1 e^{\kappa_{13} x}, \\ \mathrm{E}_{2z} = B_2 e^{\kappa_{23} x}, \end{cases} \qquad x < 0, \qquad (4)$$

where $\kappa_{j1} = \sqrt{\gamma_j^2 - k_j^2 \varepsilon_1} > 0$, $\kappa_{j3} = \sqrt{\gamma_j^2 - k_j^2 \varepsilon_3} > 0$, A_j, B_j are constants of integration. It should be noted that the constants A_j are supposed to be known (initial conditions).

Let us formulate the problem for real functions $u_1 := \mathrm{E}_{1y}$, $u_2 := \mathrm{E}_{2z}$. Thus we get $|\mathbf{E}|^2 = |\mathbf{u}|^2 = u_1^2 + u_2^2$. Inside the waveguide Σ from (2) we obtain the nonlinear system

$$\begin{cases} u_1'' + \kappa_1^2 u_1 = -k_1^2 \alpha f(|\mathbf{u}|^2) u_1, \\ u_2'' + \kappa_2^2 u_2 = -k_2^2 \alpha f(|\mathbf{u}|^2) u_2, \end{cases} \qquad (5)$$

where $\kappa_j = \sqrt{k_j^2 \varepsilon_2 - \gamma_j^2}$, $\mathbf{u} = (u_1, u_2)^\top$.

Tangential components of electromagnetic field are known to be continuous at the (open) interface. In this case the tangential components are E_{1y}, E_{2z}, H_{1z}, H_{2y} and the transmission conditions for the functions u_1 and u_2 take the form

$$[u_j]\big|_{x=0} = 0, \quad [u_j']\big|_{x=0} = 0,$$
$$[u_j]\big|_{x=h} = 0, \quad [u_j']\big|_{x=h} = 0, \qquad (6)$$

where $[w]\big|_{x=x_0} = \lim_{x\to x_0-0} w(x) - \lim_{x\to x_0+0} w(x)$. Using (3), (4), and transmission conditions (6), we obtain boundary values of the fields

$$u_j(0) = B_j, \qquad u_j'(0) = \kappa_{j3} B_j, \qquad (7)$$
$$u_j(h) = A_j, \qquad u_j'(h) = -\kappa_{j1} A_j. \qquad (8)$$

Definition 1 *Problem P is to find real pair (γ_1, γ_2) such that for given values of A_1, A_2 there are nontrivial functions u_1, u_2 such that for $0 \leqslant x \leqslant h$ functions u_1, u_2 are solutions of system* (5) *and satisfy boundary conditions* (7),(8).

3 Nonlinear integral equations

We are going to invert linear parts of the equations in (5). Let $L_j u = -k_j^2 \alpha f(|\mathbf{u}|^2) u_j$, where $L_j = \frac{d^2}{dx^2} + \kappa_j^2$. Construct Green functions for the following boundary value problems:

$$\begin{cases} L_1 G_1 = -\delta(x-s), \\ G_1'\big|_{x=0} = G_1'\big|_{x=h} = 0, \end{cases} \qquad \begin{cases} L_2 G_2 = -\delta(x-s), \\ G_2'\big|_{x=0} = G_2'\big|_{x=h} = 0. \end{cases}$$

It can be proved that the Green functions have the forms

$$G_j(x,s) = -\frac{\cos(\kappa_j \min(x,s)) \cos(\kappa_j (\max(x,s) - h))}{\kappa_j \sin(\kappa_1 h)}.$$

Using the second Green formula, conditions (7), (8) and (3), we obtain

$$\begin{cases} u_1(s) = k_1^2 \alpha \int_0^h G_1(x,s) f(|\mathbf{u}|^2) u_1 dx - \\ \qquad - \kappa_{11} A_1 G_1(h,s) - \kappa_{13} B_1 G_1(0,s), \\ u_2(s) = k_2^2 \alpha \int_0^h G_2(x,s) f(|\mathbf{u}|^2) u_2 dx - \\ \qquad - \kappa_{21} A_2 G_2(h,s) - \kappa_{23} B_2 G_2(0,s). \end{cases} \qquad (9)$$

Setting $s = h$ in (9), we get

$$A_j = k_j^2 \alpha \int_0^h G_j(x,h) f(|\mathbf{u}|^2) u_j dx + $$
$$+ \kappa_{j1} A_j \frac{\cos(\kappa_j h)}{\kappa_j \sin(\kappa_j h)} + \kappa_{j3} B_j \frac{1}{\kappa_j \sin(\kappa_j h)}. \qquad (10)$$

Setting $s = 0$ in (9), we obtain

$$B_j = k_j^2 \alpha \int_0^h G_j(x,0) f(|\mathbf{u}|^2) u_j dx + $$
$$+ \kappa_{j1} A_j \frac{1}{\kappa_j \sin(\kappa_j h)} + \kappa_{j3} B_j \frac{\cos(\kappa_j h)}{\kappa_j \sin(\kappa_j h)}. \qquad (11)$$

Using expressions (11), system (9) can be rewritten in the following form:

$$\begin{cases} u_1(s) = k_1^2 \alpha \int_0^h G_1(x,s) f(|\mathbf{u}|^2) u_1 dx + \\ + \frac{k_1^2 \alpha \kappa_{13} \cos(\kappa_1 (s-h))}{\kappa_1 \sin(\kappa_1 h) - \kappa_{13} \cos(\kappa_1 h)} \int_0^h G_1(x,0) f(|\mathbf{u}|^2) u_1 dx + \\ + \left(\cos(\kappa_1 s) + \frac{\kappa_{13} \cos(\kappa_1 (s-h))}{\kappa_1 \sin(\kappa_1 h) - \kappa_{13} \cos(\kappa_1 h)}\right) \frac{A_1 \kappa_{11}}{\kappa_1 \sin(\kappa_1 h)}, \\ u_2(s) = k_2^2 \alpha \int_0^h G_2(x,s) f(|\mathbf{u}|^2) u_2 dx + \\ + \frac{k_2^2 \alpha \kappa_{23} \cos(\kappa_2 (s-h))}{\kappa_2 \sin(\kappa_2 h) - \kappa_{23} \cos(\kappa_2 h)} \int_0^h G_2(x,0) f(|\mathbf{u}|^2) u_2 dx + \\ + \left(\cos(\kappa_2 s) + \frac{\kappa_{23} \cos(\kappa_2 (s-h))}{\kappa_2 \sin(\kappa_2 h) - \kappa_{23} \cos(\kappa_2 h)}\right) \frac{A_2 \kappa_{21}}{\kappa_2 \sin(\kappa_2 h)}. \end{cases} \qquad (12)$$

It is necessary for further actions to rewrite system (12) in an operator form.

Let $\mathrm{K}_1(x,s)$ and $\mathrm{K}_2(x,s)$ be the kernel matrices

$$\mathrm{K}_j(x,s) = \left\{K_{nm}^{(j)}(x,s)\right\}_{n,m=1}^2,$$

where $K_{jj}^{(1)}(x,s) = k_j^2 G_j(x,s)$, $K_{12}^{(j)}(x,s) = K_{21}^{(j)}(x,s) = 0$, $K_{jj}^{(2)}(x,s) = k_j^2 p_j \cos(\kappa_j (s-h)) G_j(x,0)$,

$$p_j = \frac{\kappa_{j3}}{\kappa_j \sin(\kappa_j h) - \kappa_{j3} \cos(\kappa_j h)}.$$

Introduce the matrix integral operators

$$\mathbf{K}_j\mathbf{q} = \int_0^h \mathrm{K}_j(x,s)\mathbf{q}(x)dx,$$

where $\mathbf{q} = (q_1, q_2)^\top$. Let also $\mathbf{h} = (h_1, h_2)^\top$, where

$$h_j = A_j \frac{\kappa_{j1}\big(\cos(\kappa_j s) + p_j \cos(\kappa_j(s-h))\big)}{\kappa_j \sin(\kappa_j h)}.$$

Now if we introduce two linear operators $\mathbf{N} := \alpha(\mathbf{K}_1 + \mathbf{K}_2)$, $\mathbf{N}_1 := \mathbf{K}_1 + \mathbf{K}_2$, from system (12) we get

$$\mathbf{u} = \alpha\mathbf{N}_1\big(|\mathbf{u}|^2\big) + \mathbf{h}. \tag{13}$$

We study equation (13) in $\mathbf{C}[0,h] = C[0,h] \times C[0,h]$.

Now we introduce the following notations $F(\mathbf{u}) = f\big(|\mathbf{u}|^2\big)\mathbf{u}$, $(\,\cdot\,)'_c$ is Gateaux derivative.

The following statement is valid.

Statement 1 *Let $F'_c(\mathbf{v})$ be bounded. Then there exist positive constants α, C such that $\alpha C\|\mathbf{N}_1\| < 1$ and equation* (13) *has a unique solution $\mathbf{u} = \mathbf{u}^* \in \mathbf{C}[0,h]$.*

Let $\mathbf{v}_j = (v_{j1}, v_{j2})^\top$ and $C = \sup\limits_{\mathbf{u}\in\mathbf{C}[0,h]} \|F'_c(\mathbf{u})\|$. The following estimation

$$\big\|\mathbf{N}(f\big(|\mathbf{v}_1|^2\big)\mathbf{v}_1) - \mathbf{N}(f\big(|\mathbf{v}_2|^2\big)\mathbf{v}_2)\big\| \leqslant \alpha C\|\mathbf{N}_1\|\,\|\mathbf{v}_1 - \mathbf{v}_2\|$$

is valid.

Clearly, for $q = \alpha C\|\mathbf{N}_1\| < 1$ the operator $\mathbf{N}_1$ is a contraction mapping. We choose $r > 0$ in such a way that the condition $\|\mathbf{h}\| \leqslant (1-q)r$ is fulfilled. According to lemma 2 [13, p. 382] the operator $\mathbf{N}_1$ maps the ball $\mathbf{B}_r(0) = B_r(0) \times B_r(0)$ into the same ball in $C[0,h]$, where $B_r(0) = \{x \in \mathbb{R} : \|x\| < r\}$. Then equation (13) has a unique solution $\mathbf{u} = \mathbf{u}^* \in \mathbf{B}_r(0) \subset \mathbf{C}[0,h]$. Since r can be chosen arbitrarily large, then the statement 1 is valid.

4 Dispersion equations

Combining (10), (11) and using the values of the Green functions G_1, G_2 at $s = 0, h$, we obtain the system of dispersion equations

$$\begin{cases} A_1 g_1(\gamma_1) = k_1^2\alpha\frac{Q_1(\gamma_1,\gamma_2)}{\sin(\kappa_1 h)}, \\ A_2 g_2(\gamma_2) = k_2^2\alpha\frac{Q_2(\gamma_1,\gamma_2)}{\sin(\kappa_2 h)}, \end{cases} \tag{14}$$

where

$$g_j(\gamma_j) = (\kappa_j^2 - \kappa_{j1}\kappa_{j3})\sin(\kappa_j h) - \kappa_j(\kappa_{j1} + \kappa_{j3})\cos(\kappa_j h),$$

$$Q_j(\gamma_1,\gamma_2) = \big(\kappa_{j3}\cos(\kappa_j h) - \kappa_j\sin(\kappa_j h)\big)\times \\ \times\int_0^h \cos(\kappa_j x) f(|\mathbf{u}|^2)u_j dx - \\ - \kappa_{j3}\int_0^h \cos(\kappa_j(x-h)) f(|\mathbf{u}|^2)u_j dx.$$

Setting $\alpha = 0$ in (14) and simplifying, we obtain well-known dispersion equations for a linear waveguide

$$g_1(\gamma_1) = 0, \tag{15}$$
$$g_2(\gamma_2) = 0. \tag{16}$$

5 Numerical results

In this section we chose the particular type of nonlinearity with saturation given by $f\big(|\mathbf{u}|^2\big) = \frac{|\mathbf{u}|^2}{1+\beta|\mathbf{u}|^2}$, where $\beta > 0$ is a constant, this nonlinearity is widely used in nonlinear optics [14]. For this nonlinearity $F'_c(\mathbf{v})$ is bounded and $C = \frac{13}{4\beta}$.

The following values of parameters are used for calculations in Figures 1, 2: $h = 6$ mm, $\varepsilon_1 = 1$, $\varepsilon_2 = 2$, $\varepsilon_3 = 1$, $k_1 = \sqrt{1.6}$ mm, $k_2 = 1$ mm, $A_1 = 2$ V/mm, $A_2 = 1$ V/mm.

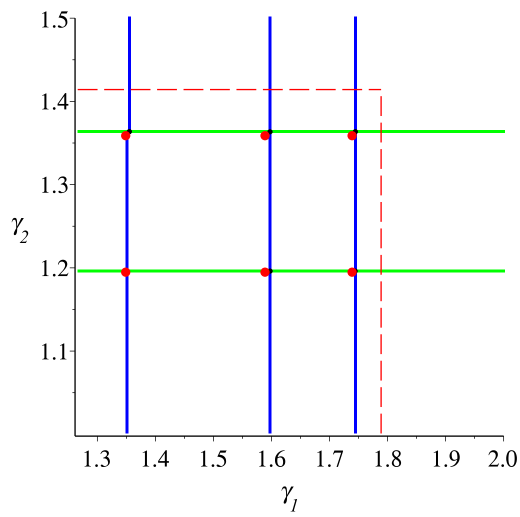

Figure 1. Solutions of the first dispersion equation (green color) and the second dispersion equation (blue color) for $\alpha = 0.01$ mm^{2}V^{-2}, $\beta = 0.9$ mm^{2}V^{-2}.

For $\alpha = 0$ we have two independent linear problems, which describe propagation of TE waves in a linear plane waveguide. Solutions of these problems can be found from dispersion equations (15), (16). Under the parameters specified above equation (15) has 3 linear eigenvalues and equation (16) has 2 linear eigenvalues.

Approximate solutions of system (14) in Figures 1, 2 are plotted. Green color corresponds to solutions of the first equation of system (14) and blue color corresponds to solutions of the second one. Black points of intersections of the blue and green curves are approximate eigenvalues of the Problem P. Red points approximate pairs of solutions of the linear problems. Red dashed lines define the domain $\tilde{\Gamma} = \big(k_1\sqrt{\max\{\varepsilon_1,\varepsilon_3\}},\ k_1\sqrt{\varepsilon_2}\big) \times \big(k_2\sqrt{\max\{\varepsilon_1,\varepsilon_3\}},\ k_2\sqrt{\varepsilon_2}\big)$. All pairs of linear solutions can exist only inside $\tilde{\Gamma}$.

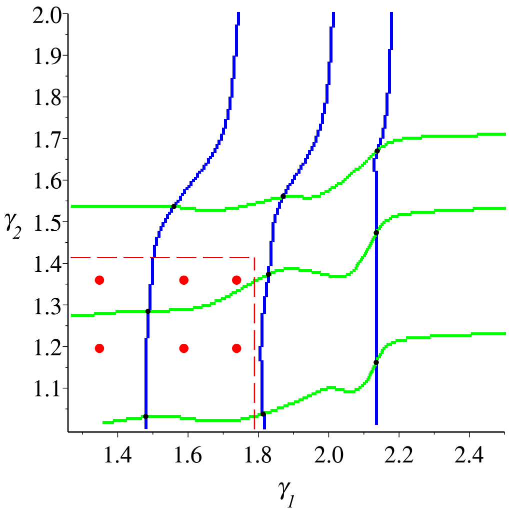

Figure 2. Solutions of the first dispersion equation (green color) and the second dispersion equation (blue color) for $\alpha = 0.09$ mm^{2}V^{-2}, $\beta = 0.08$ mm^{2}V^{-2}.

In Figure 1 the condition $\frac{13\alpha}{4\beta}\|\mathbf{N}_1\| < 1$ of statement 1 is valid and only one coupled eigenvalue corresponds to each pair of linear eigenvalues. In addition, the coupled eigenvalues are close to the corresponding pairs of linear eigenvalues. In Figure 2 $\frac{13\alpha}{4\beta}\|\mathbf{N}_1\| > 1$, there are coupled eigenvalues which do not correspond to any pairs of linear eigenvalues and there are only 2 coupled eigenvalues inside $\tilde{\Gamma}$.

Numerical results show that for $q < 1$ (for sufficiently small α and sufficiently large β) there is a clear (one-to-one) correspondence between the coupled eigenvalues and pairs of linear solutions, as illustrated by Figure 1. In this case coupled eigenvalues are close to the corresponding pairs of linear solutions. If α is sufficiently large (or β is sufficiently small) then the mentioned one-to-one correspondence is destroyed and new nonlinear solutions (coupled eigenvalues) arise, as illustrated by Figure 2.

6 Acknowledgements

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References

[1] A.D. Boardman, T. Twardowski, "Transverse-electric and transverse-magnetic waves in nonlinear isotropic waveguides," *Physical Review A*, **39**, 5, 1989, pp. 2481–2492, doi: 10.1103/PhysRevA.39.2481

[2] Chen Qin, Zi Hua Wang, "Exact dispersion relation for TM waves guided by thin dielectric films bounded by nonlinear media," *Optics letters*, **18**, 4, 1993, pp. 1–3, doi: 10.1364/OL.18.000260

[3] K.A. Yuskaeva, V.S. Serov, H.W. Schürmann, "On the theory of TM-electromagnetic guided waves in a film with nonlinear permittivity," *URSI International Symposium on Electromagnetic Theory*, 2010, pp. 439–441, doi: 10.1109/URSI-EMTS.2010.5637057

[4] H. Kiili, V. Serov, "TM-waves guided by a nonlinear film with complex permittivity," *Journal of Computational and Applied Mathematics*, **271**, 2014, pp. 339–355, doi: 10.1016/j.cam.2014.04.018

[5] O.V. Korovai, "Nonlinear s-polarized quasi-surface waves in the symmetric structure with a metamaterial core," *Physics of the Solid State* **57**, 7, 2015, pp. 1456-1462, doi: 10.1134/S1063783415070197

[6] H. W. Schürmann, V. S. Serov, "Theory of TE-polarized waves in a lossless cubic-quintic nonlinear planar waveguide," *Physical Review A*, **93**, 6, 2016, p. 063802, doi: 10.1103/PhysRevA.93.063802

[7] H. W. Schürmann, V. S. Serov, "Guided TE-waves in a slab structure with lossless cubic nonlinear dielectric and magnetic material: parameter dependence and power flow with focus on metamaterials,", *European Physical Journal D* , **73**, 9, 2019, p. 204, doi: 10.1140/epjd/e2019-100258-y

[8] Yu. G. Smirnov, D. V. Valovik, "Guided electromagnetic waves propagating in a plane dielectric waveguide with nonlinear permittivity," *Physical Review A*, **91**, 2015, p. 013840, doi: 10.1103/PhysRevA.91.013840

[9] Yu. G. Smirnov, D. V. Valovik, "Coupled electromagnetic TE-TM wave propagation in a layer with Kerr nonlinearity," *Journal of Mathematical Physics*, **53**, 12, 2012, p. 123530, doi: 10.1063/1.4769885

[10] Yu. G. Smirnov, D. V. Valovik, "Problem of nonlinear coupled electromagnetic TE-TE wave propagation," *Journal of Mathematical Physics*, **54**, 8, 2013, p. 083502, doi: 10.1063/1.4817388.

[11] D.V. Valovik, "Nonlinear coupled electromagnetic wave propagation: Saturable nonlinearities," *Wave Motion*, **60**, 2016, pp. 166–180, doi: 10.1016/j.wavemoti.2015.10.002.

[12] V.Y. Martynova, Y.G. Smirnov, "Coupled electromagnetic TE-TE wave propagation in nonlinear layer with saturated nonlinearity," *Journal of Modern Optics*, **66**, 21, 2019, pp. 2099–2113, doi: 10.1080/09500340.2019.1695004.

[13] V. A. Trenogin, "The Functional Analysis," Nauka: Moscow, 1980, 496 p.

[14] N. N. Akhmediev, A. Ankevich, "Solitons, Nonlinear Pulses and Beams," Chapman and Hall, London, 1997, 304 p.