

## Plasmonic nanoantennas and nanocavities: a transformation electromagnetics perspective

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### Abstract

Here, we apply the transformation electromagnetics technique to study the performance of plasmonic bowtie and diablo nanoantennas as well as nanocavities. The nanoparticles are illuminated by a nanoemitter and their response is evaluated in terms of the non-radiative Purcell enhancement. The influence of the polarization of the emitter, metals and aperture of the arms is analyzed. Moreover, hidden symmetries between diablo nanoantennas and nanocavities are unveiled and explained by means of the transformation electromagnetics approach.

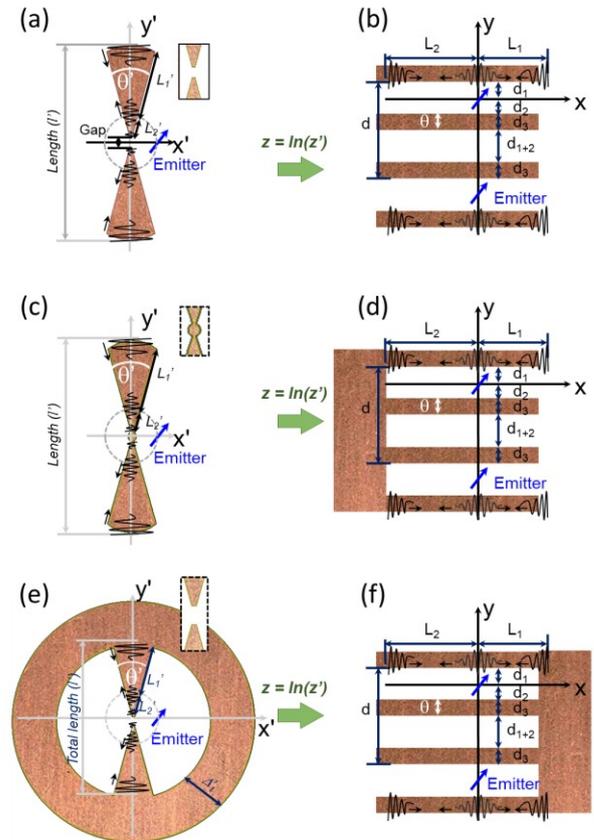
### 1 Introduction

We all make use of communications systems in a daily basis, and especially wireless applications (mobile phones, Bluetooth, smart white goods, among others). Antennas play a fundamental role in such systems where they are used as transmitters/receivers to transform electromagnetic radiation into electrical signals, and vice-versa, which can then be processed by electronic circuits[1].

In this context, the antenna's theory and design has been a prominent research field for over a century where multiple techniques have been developed to improve their performance depending on the applications (phase arrays, bowtie, parabolic antennas, among others)[2]. These methods have been the key of success of wireless communications for frequencies ranging from RF to millimeter waves.

The field of antennas evolves in a daily basis and nowadays they are becoming popular at high frequencies (near-infrared, visible, UV)[3]. This has been possible given the remarkable advance in nanofabrication techniques during the past few years, allowing to develop nanometric scaled structures made of metals and dielectrics[4]–[6]. In particular, bowtie nanoantennas and nanocavities are well regarded by the scientific community due to their high electric field concentration at their apex. This has enabled the use of antennas into other application areas at such high frequencies including enhanced spontaneous emission [7], single-molecule detection[8], spectroscopy[9] and biosensing[10].

In this realm, understanding the interaction of nanoemitters and the localized surface plasmons (LSP) resonances in such plasmonic nanostructures is becoming a prominent area of research given the strong coupling that can be achieved with such configurations[11]. Unfortunately, current design methods rely on full-wave numerical simulations where the computational burden increases when considering complex nanostructures that are made out of metals with dispersive optical properties[12]. Hence, there is a need to develop analytical tools that provide enough physical insight of the interaction between nanoemitters and plasmonic nanoparticles, to accelerate their design before being



**Figure 1.** Schematic representation of the bowtie nanoantenna (a) diabolo nanoantenna (c) and nanocavity (e) along with their corresponding transformed space (right column).

experimentally fabricated and measured.

In this context, the transformation electromagnetics method has become an important tool to study, design and understand the response of plasmonic nanoparticles[13]. This technique was first proposed and demonstrated at microwave frequencies [14] and is now being implemented in complex plasmonic nanostructures such as tripod nanoantennas[15], nanocrescents[16], nanorods[17] and metasurfaces[18], among others.

Inspired by the need of analytical methods for plasmonic nanoparticles excited by localized nanoemitters and the importance of the transformation electromagnetic technique, in this work we present our recent efforts in the study of the coupling between the radiated power of a nanoemitter and bowtie nanoantennas and nanocavities [19], [20]. Nanoparticles of length  $l' = 20$  nm are studied in terms of the non-radiated Purcell enhancement (power absorbed by the nanoparticles under dipole illumination) considering multiple metals, angles of the arms, as well as geometrical asymmetries (of high importance due to unavoidable imperfections during fabrication)[21], [22].

## 2 Analytical method

The metallic bowtie nanoantennas and nanocavities are schematically shown in the left column of Fig. 1: a bowtie with a gap = 1 nm (Fig. 1a), a bowtie with connected arms (diabolo, Fig. 1c) and a nanocavity with disconnected arms (Fig. 1e). The nanoparticles, immersed in vacuum ( $\mu = \epsilon = 1$ ), are illuminated by a point dipole located at  $(x' = 1 \text{ nm}, y' = 0)$ . After applying the transformation  $z = \ln(z'/a)$  to the nanoparticles in Fig. 1a,c,e (with  $a$  as the distance between the dipole and the center of the nanoparticles (0,0) and  $z' = x' + iy'$  and  $z = x + iy$  as the original and transformed coordinates, respectively) the final transformed geometries are those shown in Fig. 1b,d,f, respectively. As observed, multislab geometries are obtained in all cases. The multislab geometry in Fig. 1d has a metallic wall at its left-hand side corresponding to the connection between the arms of the diabolo nanoantenna in Fig. 1c. Similarly, the metallic wall at the right-hand side of the multislabs in Fig. 1d relates to the outer metallic ring of the nanocavity from Fig. 1e. Following the same approach, there are no metallic walls in either side of the transformed space in Fig. 1b since the bowtie from Fig. 1a has disconnected arms and has no outer metallic ring.

Given the underpinning properties of the transformation electromagnetic technique, the potentials and materials properties are preserved after applying the mapping, meaning that the absorbed power will be the same for the transformed geometries (Fig. 1b,d,f) and nanoparticles (Fig. 1a,c,e). This has important implications from the analytical perspective because the non-radiative Purcell enhancement can be straightforwardly obtained by simply calculating the ratio between the non-radiative power in

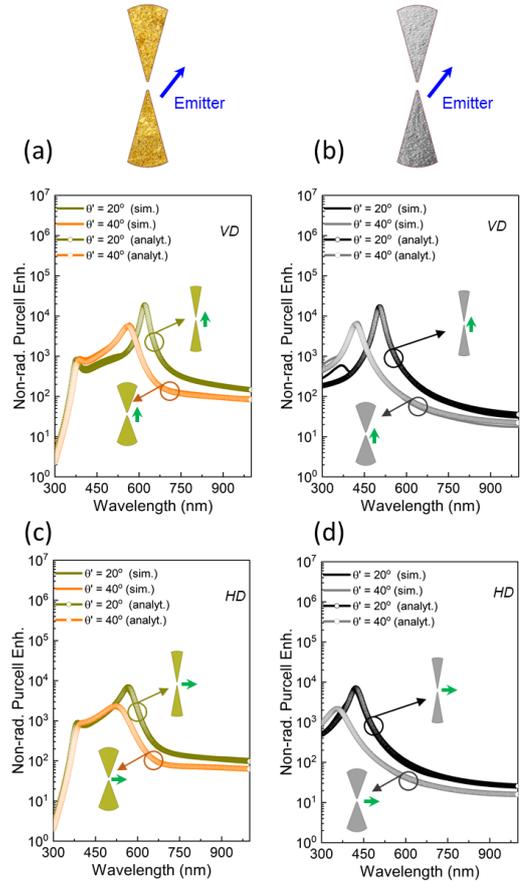
the multislab geometries and the power radiated by the nanoemitter  $P_\theta$ :

$$\overline{T}_{nr} = -(1/2P_\theta)\omega\{p_x^*E_{1x}^S(x,y) + p_y^*E_{1y}^S(x,y)\} \quad (1)$$

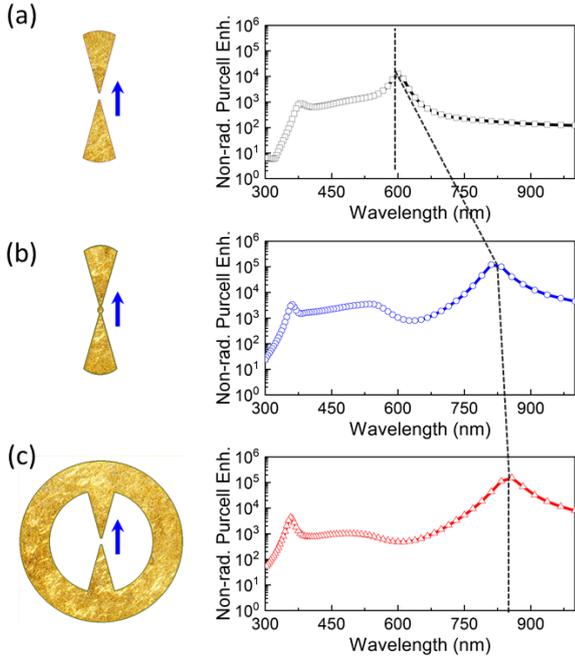
with  $\omega$  as the angular frequency,  $E_{1x,y}^S(x,y)$  and  $p_{x,y}$  as the  $x$  and  $y$  components of the electric field and dipole moment, respectively.

## 3 Results and discussion

The analytical and numerical results of the non-radiative Purcell enhancement spectra considering bowtie nanoantennas with total length  $l' = 20$  nm for different angle of the arms ( $\theta' = 20^\circ$  and  $40^\circ$ ) under vertical and horizontal polarizations are shown in Fig. 2a,b and Fig. 2c,d, respectively. Note that an excellent agreement between the results are obtained demonstrating the accuracy of the analytical model based on transformation electromagnetics.



**Figure 2.** Analytical (symbols) and numerical (lines) results of the Non-radiative Purcell enhancement spectra for bowtie nanoantennas made of gold (left column) and silver (right column) with different values of  $\theta'$  when they are illuminated with a dipole nanoemitter under vertical (a,b) and horizontal (c,d) polarization.



**Figure 3.** Analytical (symbols) and numerical (lines) results of the Non-radiative Purcell enhancement spectra nanoparticles made of gold with  $\theta' = 25^\circ$  when they are illuminated with a dipole nanoemitter under vertical polarization.

As observed, there is a clear dependence of the  $\overline{T}_{nr}$  with the angles of the arms and polarization of the nanoemitter, as expected. For instance, the LSP modes are blue shifted when increasing  $\theta'$ [19]. Since metals at optical frequencies are dispersive, it is important to select the correct metal for the design of plasmonic nanoparticles. The effect of different metals is shown in Fig. 2 where gold (Au) and silver (Ag) bowtie nanoantennas are evaluated (first and second column, respectively). As it is shown, different spectral responses are obtained depending on the selected metal and not just the magnitude of  $\overline{T}_{nr}$  is modified but also the wavelength at which the LSP modes appear. Here we evaluate only Au and Ag but we have previously shown that, for instance, using Aluminum (Al) for tripod nanoantennas can be useful for UV applications[15].

Let us now evaluate the performance of the diabolo nanoantenna and nanocavity from Fig. 1c,d considering nanoparticles made of gold and angle of the arms of  $\theta' = 25^\circ$ . The nanoparticles are illuminated with a nanoemitter placed at  $(x' = 1\text{nm}, y' = 0)$  with a vertical polarization (see schematic representations in Fig. 3). The numerical (solid lines) and analytical (symbols) results of the  $\overline{T}_{nr}$  spectra for the diabolo nanoantenna and nanocavity are shown in Fig. 3b,c, respectively. The  $\overline{T}_{nr}$  for a bowtie nanoantenna is also shown in Fig. 3a for completeness. As observed, the spectral response for the diabolo nanoantenna and nanocavity is similar with the LSP modes almost at the same wavelengths. For instance, the LSP mode of order  $n = 1$  is found at  $\sim 816$  nm and  $\sim 825$

nm for each nanoparticle, respectively, while it is completely different for the bowtie nanoantenna ( $\sim 600$  nm).

This hidden symmetry[21] between the diabolo nanoantenna and nanocavity can be explained by looking to their corresponding transformed geometries (Fig. 1d,f respectively) where it can be seen how they both share the same multislabs geometry. Finally, it is important to highlight that this symmetry is not related to the Babinet principle since i) we are considering invariant out-of-plane structures ii) the symmetry happens under the same polarization in both coordinates and iii) the diabolo nanoantenna, Fig. 1c is not the complementary version of Fig. 1e. Additional studies such as asymmetric nanoantennas and 3D nanoparticles will also be discussed during the conference[22].

## 4 Conclusion

In this work, the transformation electromagnetics technique has been applied to bowtie and diabolo nanoantennas as well as to bowtie nanocavities. With this technique, plasmonic nanoparticles are transformed into multislabs geometries which can be easily treated analytically. This has enabled us to provide physical insight of their performance when they are illuminated with a nanoemitter placed in their vicinity. The non-radiative Purcell enhancement has been calculated for all the nanoparticles demonstrating a clear dependence on the selected metals, polarization of the nanoemitter and angle of the arms, among other parameters. Moreover, hidden symmetries between the diabolo nanoantenna and nanocavity have been unveiled and explained in terms of the transformation electromagnetic approach where both nanoparticles share the same transformed space. These results can help to understand the performance of plasmonic nanoparticles and efficiently design them before being fabricated and experimentally demonstrated, reducing computational burden and experimental costs.

## 5 Acknowledgements

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