



Outage Probability of Two-Way Relaying Systems Over Mixed Fluctuating Two-Ray and Nakagami- m Fading Channels

Jiayi Zhang^{*(1)}, Yongshun Zhang⁽¹⁾, and Bo Ai⁽²⁾

(1) School of Electronic and Information Engineering, Beijing Jiaotong University, Beijing 100044, China

(2) State Key Laboratory of Rail Traffic Control and Safety, Beijing Jiaotong University, Beijing 100044, China

Abstract

Two-way relaying systems can significantly save time resource and increase the throughput for wireless communication systems. In this paper, we carry out the performance analysis of two-way relaying systems over mixed fluctuating two-ray (FTR) and Nakagami- m fading channels. The FTR fading promises a good fit for small-scale experimental data in millimeter wave communications. More specifically, we derive novel and exact analytical expressions for outage probability (OP) and simple asymptotic OP at high signal-to-noise-ratio regimes to show important physical insights into the impact of parameters on the system performance. It is interesting to find that larger values of fading severity parameters of both links help to decrease the OP. Finally, the correctness of our derived analytical results is validated by Monte Carlo simulations.

1 Introduction

Recently, two-way relaying systems have received a lot of attention for bringing huge effective system throughput in wireless communications. With the aid of a relay node, two-way relaying can significantly save the time resource. In contrast to half-duplex relaying scheme, two-way relaying enables the source and destination communicate within only two time slots. The performance of two-way interference-limited amplify-and-forward (AF) relaying systems over independent, non-identically distributed Nakagami- m fading channels was investigated in [1]. Moreover, performance analysis and relay selection of two-way hybrid terrestrial-satellite relaying systems were proposed in [2], which analyzes the performance of systems over Nakagami- m and κ - μ shadowed fading channels.

The FTR channel fading model recently proposed in [3] has been proved to be very useful, since it is in good agreement with the experimental data of millimeter wave (mmWave) communications. Therefore, considerable research has been carried out on the FTR fading channel. For example, exact probability distribution function (PDF) and cumulative distribution function (CDF) for the FTR distribution with arbitrary parameters were proposed in [4]. To the best of our knowledge, however, the mmWave bands have not been considered in the two-way relaying system-

s. To this end, for the successful deployment of mmWave bands in two-way relaying systems, a complete understanding of its performance when operating under FTR fading channels becomes essential.

Motivated by the above discussion, in this paper we present novel analytical expressions for outage probability (OP) of two-way relaying systems over mixed FTR and Nakagami- m fading channels. Moreover, simple asymptotic expressions for the OP are derived to obtain important engineering insights in the high-SNR regime. Finally, our results can generalize most of previous results in the literature.

2 Two-Way Relaying Systems

2.1 System Model

Similar to [2], let us consider two transmitter nodes A and B communicating with each other through an AF relay R . Due to poor communication quality in the direct link, we adopt a relay R to assist the transmission of links as $A \rightarrow R \rightarrow B$ and $B \rightarrow R \rightarrow A$. All nodes are equipped with one antenna. The transmission process between A and B is divided into two time phases. Specifically, in the first phase, both A and B transmit symbols to the relay R simultaneously. In the second phase, the relay R processes the received signal to forward the combined symbols to both A and B . We assume that the channel state information is completely known at each node, and perfect synchronization between A , B and R is achieved. Then, the instantaneously received SNRs at A and B can be given by [5]

$$\gamma_A = \frac{P_2 Q \gamma_1 \gamma_2}{(P_1 + Q) \gamma_1 + P_2 \gamma_2}, \quad (1)$$

$$\gamma_B = \frac{P_1 Q \gamma_1 \gamma_2}{(P_2 + Q) \gamma_2 + P_1 \gamma_1}, \quad (2)$$

where $\gamma_1 = |h_1|^2/N_0$, $\gamma_2 = |h_2|^2/N_0$, and N_0 denotes the variance of the additive white Gaussian noise (AWGN) at all nodes. On the other hand, We use h_1 and h_2 to denote the channel coefficients of $A \rightarrow R$ and $B \rightarrow R$ links. P_1 , P_2 , and Q denote the transmit power at A , B , and R , respectively. Without loss of generality, we introduce power coefficients as $p_d \triangleq \frac{P_1+Q}{P_2Q}$, $p_s \triangleq \frac{P_2+Q}{P_1Q}$, $b_q \triangleq 1/Q$. Then, we can rewrite

(1) and (2) as

$$\gamma_A = \frac{\gamma_1 \gamma_2}{p_d \gamma_1 + b_q \gamma_2}, \quad (3)$$

$$\gamma_B = \frac{\gamma_1 \gamma_2}{b_q \gamma_1 + p_s \gamma_2}. \quad (4)$$

2.2 Channel Model

The Nakagami- m fading is assumed for the $A \leftrightarrow R$ link. The instantaneous SNR of the $A \leftrightarrow R$ link, γ_1 , is a gamma distributed RV. Then the probability distribution function (PDF) of γ_1 is given by [2, Eq. (7)]

$$f_{\gamma_1}(\gamma) = \frac{m_1^{m_1}}{\bar{\gamma}_1^{m_1} \Gamma(m_1)} \gamma^{m_1-1} \exp\left(-\frac{m_1 \gamma}{\bar{\gamma}_1}\right), \quad (5)$$

where $\Gamma(\cdot)$ is the gamma function [6, Eq. (8.310.1)] and m_1 is the Nakagami- m fading parameter. $\bar{\gamma}_1 = \sigma_{h_1}^2 \gamma_0$ is the average SNR of the $A \leftrightarrow R$ links, and $\sigma_{h_1}^2 = E[|h_1|^2]$ is the expectation operator. Moreover, we introduce $\gamma_0 \triangleq P_1/N_0$ as the average transmit SNR of the $A \rightarrow R$ link, and introduce $\gamma_0 \triangleq Q/N_0$ for the $R \rightarrow A$ link, respectively. Hence, the CDF of γ_1 can be written as

$$F_{\gamma_1}(\gamma) = 1 - \frac{\Gamma(m_1, (m_1/\bar{\gamma}_1)\gamma)}{\Gamma(m_1)}, \quad (6)$$

where $\Gamma(\cdot, \cdot)$ is the upper incomplete gamma function [7, Eq. (8.350.2)]. Furthermore, the $B \leftrightarrow R$ link is modeled as the FTR distribution, which provides better fit than other channel models with small-scale fading measurements in mmWave communications [3, 4]. The PDF of the instantaneous SNR of $S \leftrightarrow R$ links, γ_2 , is given as [4, Eqs. (6)]

$$f_{\gamma_2}(\gamma) = \frac{m_2^{m_2}}{\Gamma(m_2)} \sum_{j=0}^{\infty} \frac{K^j d_j}{j!} f_G(\gamma; j+1, 2\sigma^2), \quad (7)$$

where $f_G(\gamma; j+1, 2\sigma^2)$ and d_j is given in [4, Eqs. (8-9)]. K denotes the ratio of the average power of the dominant wave to the remaining diffuse multipath and m_2 is the fading severity parameter. Moreover, $P(\cdot)$ is the Legendre function of the first kind [7, Eq. (8.702)], while Δ is a variable from 0 to 1, representing the correlation between the two dominant waves. Furthermore, $\bar{\gamma}_2 = \sigma_{h_2}^2 \gamma_0$ represents the average SNR of the $B \leftrightarrow R$ links. Similarly, $\sigma_{h_2}^2 = E[|h_2|^2]$ is the variance of the channel coefficient h_2 , where $\gamma_0 \triangleq P_2/N_0$ and $\gamma_0 \triangleq Q/N_0$ denotes the average transmit SNR of the $B \rightarrow R$ link and of the $R \rightarrow B$ link, respectively. Then, the CDF of γ_2 is given as

$$F_{\gamma_2}(\gamma) = \frac{m_2^{m_2}}{\Gamma(m_2)} \sum_{j=0}^{\infty} \frac{K^j d_j}{j!} F_G(\gamma; j+1, 2\sigma^2), \quad (8)$$

where

$$F_G(\gamma; j+1, 2\sigma^2) \triangleq \frac{1}{\Gamma(j+1)} \gamma(j+1, \frac{\gamma}{2\sigma^2}), \quad (9)$$

where $\gamma(\cdot, \cdot)$ is the lower incomplete gamma function [7, Eq. (8.350.1)].

Table 1. Required Terms of P_{o,γ_A} for Truncation Error Smaller Than 10^{-3} With Different m_1, m_2, K and Δ ($\gamma_{th}=2$ dB and $\bar{\gamma}_1 = \bar{\gamma}_2$)

Average SNR $\bar{\gamma}_1$ [dB]	5	12.5	20	30
$m_1 = 5, m_2 = 5, K = 5, \Delta = 0.35$	21	18	5	2
$m_1 = 2, m_2 = 3, K = 3, \Delta = 0.1$	23	16	14	3
$m_1 = 3, m_2 = 15, K = 20, \Delta = 0.48$	47	34	33	1

Table 2. Required Terms of P_{o,γ_B} for Truncation Error Smaller Than 10^{-3} With Different m_1, m_2, K and Δ ($\gamma_{th}=3$ dB and $\bar{\gamma}_1 = \bar{\gamma}_2$)

Average SNR $\bar{\gamma}_1$ [dB]	5	12.5	20	30
$m_1 = 3, m_2 = 1, K = 10, \Delta = 0.2$	57	72	76	75
$m_1 = 5, m_2 = 5, K = 5, \Delta = 0.35$	23	31	27	26
$m_1 = 1, m_2 = 10, K = 20, \Delta = 0.35$	51	59	60	60

3 Performance Analysis

3.1 Outage Probability

The outage probability $P_o(\gamma_{th})$ is defined as the probability that the instantaneous SNR is below a given threshold, and it can be written as $P_o(\gamma_{th}) = \Pr(\gamma < \gamma_{th})$. Therefore, we can derive the OP at A as

$$P_{o,\gamma_A}(\gamma_{th}) = F_{\gamma_A}(\gamma_{th}), \quad (10)$$

$$P_{o,\gamma_B}(\gamma_{th}) = F_{\gamma_B}(\gamma_{th}), \quad (11)$$

where $F_{\gamma_A}(\gamma_{th})$ and $F_{\gamma_B}(\gamma_{th})$ represent the CDFs at A and B in the case of instantaneous SNR, respectively. Then the exact OP is given by the following Lemma.

Lemma 1. *The exact OP at A and B of two-way relaying systems are given as (12) and (13) at the bottom of the next page, where m_1 is a positive integer, $\Phi_1 \triangleq m_1 b_q / \bar{\gamma}_1$, $\Phi_2 \triangleq p_d / 2\sigma^2$, $\Psi_1 \triangleq m_1 p_s / \bar{\gamma}_1$, $\Psi_2 \triangleq b_q / 2\sigma^2$, $c_j \triangleq \frac{m_2^{m_2} K^j d_j}{\Gamma(m_2) \Gamma(j+1)}$, and $K_\nu(\cdot)$ denotes the ν^{th} -order modified Bessel function of the second kind [8, Eq. (9.6.2)].*

Proof. Please see Appendix. \square

Tables I and II present the required terms of infinite series in (12) and (13) for a required convergence. Although there are infinite terms in the formulas, it requires a small number of terms to reduce the error to smaller than 10^{-3} for all considered cases. As shown in the Tables, the truncation error is related to different channel fading parameters.

Considering Rician shadowed fading model (by setting $\Delta = 0$), (12) and (13) can reduce to the ones for mixed Nakagami- m and Rician shadowed fading channels in [2, Eqs. (15-16)]. Moreover, (12) and (13) can reduce to the OP of two-way AF relaying systems over Nakagami- m (by setting $K \rightarrow \infty$ and $\Delta = 0$) fading channels in [9, Eq. (15)].

3.2 Asymptotic Outage Probability

In order to reveal more engineering insights, we elaborate on the asymptotic high-SNR regime.

Lemma 2. *With high-SNR regime, the asymptotic OP of two-way relaying systems at A and B can be expressed as*

$$\begin{aligned}
P_{o,\gamma_A}^\infty &\approx 1 - \sum_{i=0}^2 \sum_{k=0}^{m_1-1} \frac{m_1^k b_q^k}{i!k! \bar{\gamma}_1^k} (-\Phi_1 \gamma)^i \gamma^k + \sum_{i=0}^2 \sum_{j=0}^\infty \sum_{r=0}^{m_1-1} \\
&\times \binom{m_1-1}{r} \left(\frac{m_1}{\bar{\gamma}_1}\right)^{m_1-r-1} \frac{c_j \Gamma(r+1) b_q^{m_1-r-1}}{i! \Gamma(m_1)} (-\Phi_1 \gamma)^i \\
&\times \gamma^{m_1-r-1} - \sum_{i=0}^2 \sum_{j=0}^\infty \sum_{t=0}^j \sum_{s=0}^t \sum_{r=0}^{m_1-1} \binom{m_1-1}{r} \binom{t}{s} \\
&\times \left(\frac{m_1}{\bar{\gamma}_1}\right)^{m_1-\frac{r-s+1}{2}} \frac{c_j \Gamma(|r-s+1|) b_q^{m_1+\frac{3s-r-1}{2}}}{i!t! \Gamma(m_1)} \Phi_2^{t+\frac{r-s+1}{2}} \\
&\times (-\Phi_1 + \Phi_2) \gamma^i (\Phi_1 \Phi_2)^{-\frac{|r-s+1|}{2}} \gamma^{m_1+t-|r-s+1|}, \quad (14)
\end{aligned}$$

$$\begin{aligned}
P_{o,\gamma_B}^\infty &\approx 1 - \sum_{i=0}^2 \sum_{j=0}^\infty \sum_{r=0}^j \sum_{k=0}^{m_1-1} \sum_{v=0}^k \binom{j}{r} \binom{k}{v} \\
&\times \frac{c_j \Gamma(|r-v+1|) b_q^{j-\frac{r-v-1}{2}}}{i!j!k! (2\sigma^2)^{j-\frac{r-v-1}{2}}} (-\Psi_1 + \Psi_2) \gamma^i \\
&\times (\Psi_1 \Psi_2)^{-\frac{|r-v+1|}{2}} (\Psi_1)^{k+\frac{r-v+1}{2}} \gamma^{j+k+1-|r-v+1|}. \quad (15)
\end{aligned}$$

Proof. For high SNRs, according to [8, Eq. (9.6.6)] and [8, Eq. (9.6.9)], we can express $K(\cdot)$ as

$$K_{r-v+1} \left(\sqrt{\frac{2m_1 p_s b_q \gamma^2}{\bar{\gamma}_1 \sigma^2}} \right) \approx \frac{(|r-v+1|)}{2} \left(\frac{m_1 p_s b_q \gamma^2}{2\bar{\gamma}_1 \sigma^2} \right)^{|r-v+1|}. \quad (16)$$

According to [7, Eq. (1.211.1)], the exponential function e^x can be represented by the main decisive terms (here we use 3 smallest exponents). After some mathematical simplifications, (14) can be obtained. Following similar steps, we can obtain (15) to finish the proof. \square

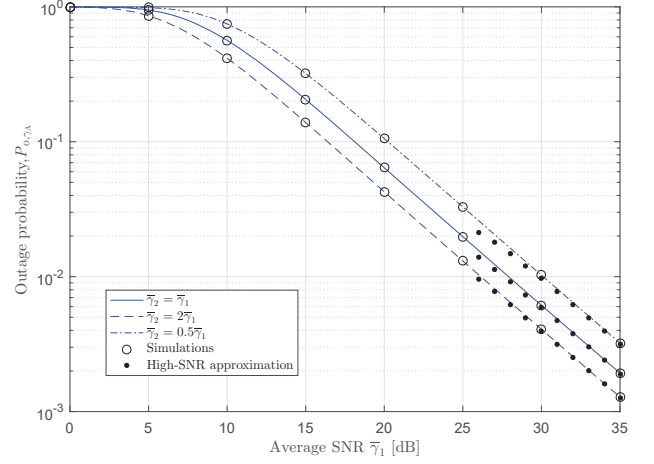


Figure 1. The OP of transmitter node A against the average SNR $\bar{\gamma}_1$ for different values of $\bar{\gamma}_2$ ($m_1 = m_2 = 1$, $K = 10$, $\Delta = 0.2$ and $\gamma_{hh} = 3$ dB).

We can find several important engineering insights from the above results. For example, larger values of m_1 , m_2 and larger average SNR of the links help to decrease the OP. This is because both light shadowing (larger m_1 and m_2) and high SNRs reduce the effects of fading.

4 Numerical Results

Figure 1 depicts the analytical, simulated, and high-SNR approximate OP at transmitter node A for different values of $\bar{\gamma}_2$. It can be clearly seen that simulation and analytical curves fit well, which validates the accuracy of our previous derived results. Moreover, the approximations are quite tight with the exact OP in the high-SNR regime. Increasing the SNR at one link (such as $\bar{\gamma}_1$), the OP decreases. The impact of different values of channel parameters m_1 and m_2 on the OP at B is evaluated in Fig. 2. With m_1 and/or m_2 increasing, it is obviously seen that the P_{o,γ_B} decreases.

5 Conclusion

In this paper, we derive exact analytical expressions of OP to investigate the performance of two-way relaying systems over mixed FTR and Nakagami- m fading channels. To

$$\begin{aligned}
P_{o,\gamma_A}(\gamma) &= 1 - \sum_{k=0}^{m_1-1} \frac{m_1^k b_q^k}{k! \bar{\gamma}_1^k} \exp(-\Phi_1 \gamma) \gamma^k + \sum_{j=0}^\infty \sum_{r=0}^{m_1-1} \binom{m_1-1}{r} \left(\frac{m_1}{\bar{\gamma}_1}\right)^{m_1-r-1} \frac{c_j \Gamma(r+1) b_q^{m_1-r-1}}{\Gamma(m_1)} \exp(-\Phi_1 \gamma) \gamma^{m_1-r-1} \\
&- 2 \sum_{j=0}^\infty \sum_{t=0}^j \sum_{s=0}^t \sum_{r=0}^{m_1-1} \binom{m_1-1}{r} \binom{t}{s} \left(\frac{m_1}{\bar{\gamma}_1}\right)^{m_1-\frac{r-s+1}{2}} \frac{c_j b_q^{m_1+\frac{s-r-1}{2}}}{t! \Gamma(m_1)} \Phi_2^{t+\frac{r-s+1}{2}} \exp(-(\Phi_1 + \Phi_2) \gamma) \gamma^{m_1+t} K_{r-s+1} \left(2\gamma \sqrt{\Phi_1 \Phi_2}\right), \quad (12)
\end{aligned}$$

$$P_{o,\gamma_B}(\gamma) = 1 - 2 \sum_{j=0}^\infty \sum_{r=0}^j \sum_{k=0}^{m_1-1} \sum_{v=0}^k \binom{j}{r} \binom{k}{v} \frac{c_j b_q^{j-\frac{r-v-1}{2}}}{j!k! (2\sigma^2)^{j-\frac{r-v-1}{2}}} \Psi_1^{k+\frac{r-v+1}{2}} \exp(-(\Psi_1 + \Psi_2) \gamma) \gamma^{j+k+1} K_{r-v+1} \left(2\gamma \sqrt{\Psi_1 \Psi_2}\right). \quad (13)$$

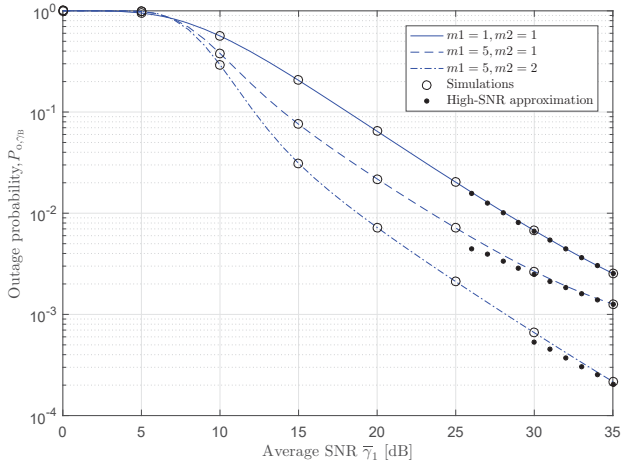


Figure 2. The OP of transmitter node B against the average SNR $\bar{\gamma}_1$ for different values of m_1 and m_2 ($K = 10$, $\Delta = 0.2$, $\bar{\gamma}_1 = \bar{\gamma}_2$ and $\gamma_h = 3\text{dB}$).

obtain important physical insights and simplify the calculation, closed-form expressions for high-SNR regime have been presented. Our results reveals the relationship between channel parameters and system performance. For instance, OP performance can be improved by increasing the values of m_1 and/or m_2 . Furthermore, the OP performance of the two-way relaying systems can also be improved by increasing the values of the transmit power at A , B and/or the average SNR at either link. Ultimately, the presented results are quite useful for satisfying the performance requirements of a practical two-way relaying mmWave system.

6 Appendix

With the help of (5), we can express the $F_{\gamma_A}(\gamma)$ as

$$\begin{aligned}
 F_{\gamma_A}(\gamma) &= \int_0^\infty Pr(\gamma_A \leq \gamma | \gamma_1) f_{\gamma_1}(\gamma_1) d\gamma_1 \\
 &= \int_0^{b_q\gamma} Pr\left(\gamma_2 \geq \frac{p_d\gamma\gamma_1}{\gamma_1 - b_q\gamma} \middle| \gamma_1\right) f_{\gamma_1}(\gamma_1) d\gamma_1 \\
 &+ \int_{b_q\gamma}^\infty Pr\left(\gamma_2 \leq \frac{p_d\gamma\gamma_1}{\gamma_1 - b_q\gamma} \middle| \gamma_1\right) f_{\gamma_1}(\gamma_1) d\gamma_1 \\
 &= F_{\gamma_1}(b_q\gamma) + \underbrace{\int_{b_q\gamma}^\infty F_{\gamma_2}\left(\frac{p_d\gamma\gamma_1}{\gamma_1 - b_q\gamma}\right) f_{\gamma_1}(\gamma_1) d\gamma_1}_{A_1}. \quad (17)
 \end{aligned}$$

Substituting (6) into (17), and with the help of [7, Eq. (8.352.2)], $F_{\gamma_1}(b_q\gamma)$ can be solved. According to (5), (8) and (9), we can simplify the integral A_1 as

$$\begin{aligned}
 A_1 &= \frac{m_1^{m_1-1} m_2^{m_2}}{\bar{\gamma}_1 \Gamma(m_1) \Gamma(m_2)} \bar{\gamma}_1 \exp\left(-\frac{m_1 b_q \gamma}{\bar{\gamma}_1}\right) \sum_{j=0}^{\infty} \sum_{r=0}^{m_1-1} \\
 &\times \frac{K^j d_j}{\Gamma(j+1) \Gamma(j+1)} (b_q \gamma)^{m_1-r-1} \binom{m_1}{r} \\
 &\times \underbrace{\int_0^\infty x^r \gamma \left(j+1, \frac{p_d \gamma (x + b_q \gamma)}{2\sigma^2 x}\right) \exp\left(-\frac{m_1 x}{\bar{\gamma}_1}\right) dx}_{A_2}.
 \end{aligned}$$

With the help of [7, Eq. (8.356.3)], [7, Eq. (3.351.3)], [7, Eq. (8.352.2)] and [7, Eq. (3.471.9)], A_2 can be easily solved. Then P_{o,γ_A} is obtained after some simplifications. Similar to (17), the CDF of γ_B can be expressed as

$$\begin{aligned}
 F_{\gamma_B} &= F_{\gamma_2}(b_q\gamma) + \int_{b_q\gamma}^\infty F_{\gamma_1}\left(\frac{p_s\gamma\gamma_2}{\gamma_2 - b_q\gamma}\right) f_{\gamma_2}(\gamma) d\gamma_2 \\
 &= 1 - \underbrace{\int_{b_q\gamma}^\infty \bar{F}_{\gamma_1}\left(\frac{p_s\gamma\gamma_2}{\gamma_2 - b_q\gamma}\right) f_{\gamma_2}(\gamma_2) d\gamma_2}_{B_1}, \quad (18)
 \end{aligned}$$

With the help of [7, Eq. (3.471.9)], the CDF of γ_B can be obtained with some mathematical simplifications.

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