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The Full-Wave Alternative to Eddy-Current Solvers: on a Low-Frequency and Dense-Discretization Stable PMCHWT Equation for Dielectric and Conductive Media

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Among the integral equations for the modeling of the electromagnetic scattering by dielectric and conductive objects, the Poggio-Miller-Chang-Harrington-Wu-Tsai (PMCHWT) formulation represents a favorable choice due to its superior accuracy. Nevertheless, given its first-kind nature, its discretization leads to an ill-conditioned system of equations, resulting in an increase in complexity of its solution process with increasing number of degrees of freedom. In addition, the PMCHWT equation is also plagued by the well-known low-frequency breakdown, as well as by further conductivity instabilities. The severe ill-conditioning limits the use of the stand-alone PMCHWT equation in many real-case scenarios, often crucial from the industrial point of view. This is the case for example of low-frequency electromagnetic compatibility and interference (EMC/EMI) circuit simulations, of pivotal importance in the design phase of any electronic equipment, which are nowadays often based on quasi-static, eddy-current approximations of Maxwell's system and require *ad-hoc* solvers. Since they rely on approximate physics, these solvers cannot be applied to multi-scale scenarios, where different approximations would need to co-exist, in which case full-wave, exact modelling is strictly required.

To answer this need, we propose the first mesh refinement preconditioning strategy for the PMCHWT equation that can be applied to dielectric and conductive media and is valid in the whole low-frequency regime, including the eddy-current one. Our stabilization cures several conditioning issues affecting the PMCHWT equation, resulting in a frequency and conductivity stable integral equation (i.e., an overall well conditioned perturbation of the identity), immune from the dense-discretization breakdown. Moreover, we avoid the loss of solution accuracy typically occurring at extremely low frequency in finite-precision arithmetic for the most common types of excitations, including inductive and capacitive ones. This was achieved through the accurate asymptotic analysis and rescaling of the quasi-Helmholtz components of the system, by means of primal and dual quasi-Helmholtz projectors, which offer crucial advantages with respect to the standard Loop-Star decomposition. Then, by multiplying the quasi-Helmholtz-stabilized matrix by its dual counterpart, we also achieved the mesh-refinement stability, following from the fact that the PMCHWT operator, when properly discretized, is a valid preconditioner for itself.

The theoretical analysis is complemented and corroborated by a numerical study of the proposed formulation, applied to simply and multiply connected geometries. Figures 1a and 1b show the comparison between the standard PMCHWT equation and this work in terms of condition number for a wide range of frequencies and conductivities, testifying both of the strong limitations of the first, unusable in the majority of situations represented, and the stability of the second one, whose condition number is bounded and constant in the different frequency-conductivity regimes studied. The dense-discretization stability of the formulation is assessed in fig. 1c, where the condition number as a function of the inverse mesh refinement parameter h (average mesh edge length) is compared with the one of the quasi-Helmholtz-stabilized formulation.

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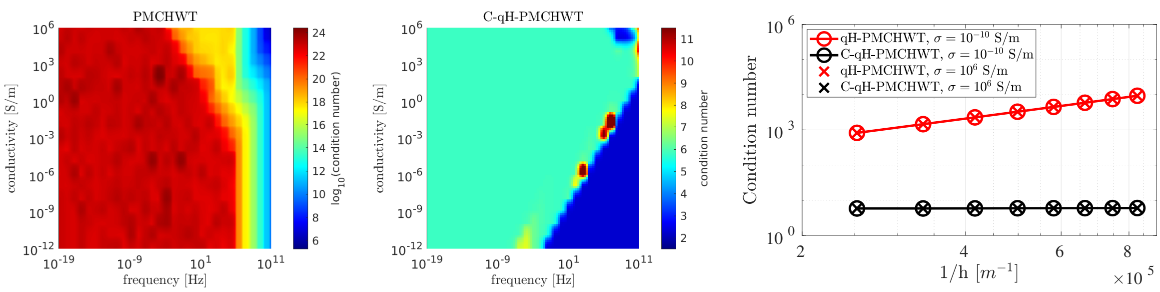

(a) **(b)** **(c)**

Fig. 1. Condition number of (a) the PMCHWT formulation (in logarithmic scale) and (b) this work. (c) Condition number as a function of the inverse mesh refinement parameter h. The geometry is a sphere with radius 10 µm.