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Data-Driven Representations of Ion-Kinetic Distribution Functions

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Abstract

Collisionless processes, such as wave particle interactions, are key to understanding the energy transfer in plasma environments. While collisional interactions are known to result in a distribution of particle velocities close to a thermal Maxwell-Boltzmann distribution, wave-particle interactions produce distribution functions that may diverge significantly from thermal equilibrium. Correct measurement and representation of non-thermal features is key to understanding the collisionless wave-particle interactions and how they shape plasma distributions. Here, we present and expand on methods (Polynomial interpolation, radial basis functions) used to approximate ion distribution functions observed by Parker Solar Probe. These non-parametric representations of the observed distributions can be used to understand instability growth rates and resonant heating of the solar wind plasma.

1 Introduction

The dissipation of turbulence is important in heating and accelerating astrophysical plasmas [1]. As a result of the collisionless nature of many plasma environments, viscous interactions, which are typically invoked in the dissipation of hydrodynamic turbulence, cannot be the primary means of transferring turbulent energy into particle thermal motion, i.e. heat [1]. Wave-particle interactions occur widely in turbulent plasma environments, such as the solar wind, and may serve as a fundamental pathway to plasma heating and turbulent dissipation at ion-kinetic scales in the solar wind [1, 2, 3, 4, 5].

In practice, kinetic descriptions of plasmas are obtained via a probabilistic *distribution function*, $f_s(\mathbf{x},\mathbf{v},t)$ defined for a particle species s (e.g. protons or electrons), in a six-dimensional *phase-space* comprising velocity and position coordinates [1]. The collisionless evolution of the distribution function occurs as:

$$\frac{\partial f_s}{\partial t}+\mathbf{v}\cdot\nabla f_s+\frac{\mathbf{F}}{m_s}\cdot\frac{\partial f_s}{\partial \mathbf{v}}=\left(\frac{\partial f_s}{\partial t}\right)_{\mathrm{coll}}, \tag{1}$$

with e.g. electromagnetic forces $\mathbf{F}=q_s(\mathbf{E}+\mathbf{v}\times\mathbf{B})$, which is known as the collisionless Boltzmann or Vlasov equation. In collisional plasmas, interactions between individual particles are frequent, leading the distribution of particle velocities to relax to a Maxwellian distribution [1]:

$$f(v)\;d^3v=\left(\frac{m}{2\pi kT}\right)^{3/2}e^{-\frac{mv^2}{2kT}}\;d^3v, \tag{2}$$

where m is the particle mass, $k_B$is Boltmzann's constant, and $T=\frac{mw^2}{2k_B}$ relates temperature to the thermal speed w. However, the lack of collisional processing in many space plasma environments enables particle distribution functions to deviate significantly from Maxwellian distributions[1, 3, 4].

Canonically, non-thermal ion distributions in heliospheric plasma environments are represented through sums of several individual particle populations [1, 6]: e.g., two anisotropic biMaxwellians with a drift v_D parallel the background magnetic field [1] representing beam and core populations

$$f(v_\perp,v_\parallel)= \tag{3}$$
$$\frac{n_c}{\pi^{3/2}w_{c,\perp}^2 w_{c,\parallel}}\exp\left[-\frac{v_\perp^2}{w_{c,\perp}^2}-\frac{v_\parallel^2}{w_{c,\parallel}^2}\right]$$
$$+\frac{n_b}{\pi^{3/2}w_{b,\perp}^2 w_{b,\parallel}}\exp\left[-\frac{v_\perp^2}{w_{b,\perp}^2}-\frac{(v_\parallel-v_D)^2}{w_{b,\parallel}^2}\right].$$

Even though particle populations often clearly deviate from Maxwellian distributions, the space plasma physics community often approximates populations with Maxwellian functions or, more accurately, extensions of the Maxwellian function to account for anisotropic temperatures, (e.g. bi-Maxwellian approximations), and/or multiple particle populations (e.g. sets of superposed drifting bi-Maxwellian plasma populations). While such extensions capture non-thermal features of ion distributions, there is no a priori physical reason to parameterize non-thermal in terms of anisotropic, drifting, bi-Maxwellian populations.

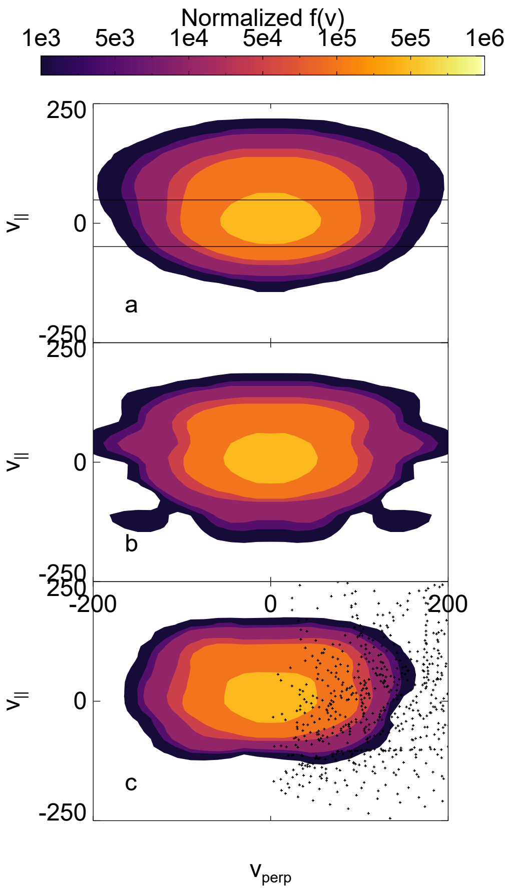

Figure 1. a) Drifting biMaxwellian fit. b) Hermite polynomial approximation. c) Radial Basis Function approximation. The solid lines in a) show the parallel thermal speed. The dots in c) show the locations of the SPAN energy bins

While the commonly used drifting-biMaxwellian model provides reasonable fits to proton distributions in the solar wind [1, 6, 4], such approximations may not capture every aspect of the measured kinetic distribution. Only a handful of previous studies have applied non-parametric representations to observed distribution functions [8, 9, 10]. The application of polynomial based and data-driven representations to observed proton distributions in the Parker Solar Probe (PSP) allow us to make progress in understanding of wave-particle interactions occurring in plasma environments.

2 Nonparametric Representations

Approximating observed distribution functions using the drifting biMaxwellian function defined in Equation 3 requires performing a nonlinear parametric fit to solve for the thermal speeds, population drift, and centroid, i.e. bulk speed, of the distribution. These parametric fits have relatively ill-defined error, and are highly subject to the fitting conditions and regularization applied. In contrast, nonparametric approximations can be obtained using relatively direct optimization methods. We highlight two methods, both Polynomial approximation using Hermite polynomials [10, 5] and the use of Radial Basis Functions [11].

2.1 Polynomial Approximation

Polynomial interpolation can approximate the measured ion distribution function through fitting the observed distribution function $f(v_\perp, v_\parallel)$ to a set of orthogonal-basis polynomials. A Hermite decomposition has previously been successfully implemented to study kinetic plasma physics in the solar wind and magnetosphere [10, 5], while previous studied have used other orthogonal basis functions [8, 9]. Following [5] we perform a linear least square fit to the observed distribution functions using the Hermite polynomials H_n, and Hermite functions ϕ_m

$$f^H(v_\perp, v_\parallel) = \sum_{m,n} g_{mn}\phi_m(v_\perp/v_{\perp th})\phi_n(v_\parallel/v_{\parallel th}) \quad (4)$$

$$H_n(v) = (-1)^n e^{v^2}\frac{d^n}{dx^n}e^{-v^2} \quad (5)$$

$$\phi_m = \frac{H_m(v)}{\sqrt{2^m\pi^{1/2}m!}}e^{-v^2}. \quad (6)$$

The Hermite coefficients g_{mn} can be directly obtained through SVD estimate of the pseudoinverse. Weighted least squares fits can be adopted depending on the noise estimates of the measured distribution.

2.2 Radial Basis Function

The radial basis function (RBF) approach is similar to the polynomial interpolation method, however, instead of orthogonal polynomial basis functions, the distribution function is modeled as the sum of a set of radial basis functions [11]. The RBF method requires choice of a basis function, which we choose to be an isotropic bi-Maxwellian

$$\psi = \frac{1}{\sqrt{\pi^3 w_{RBF}^3}}\exp\left[-\frac{v_\perp^2}{w_{RBF}^2} - \frac{v_\parallel^2}{w_{RBF}^2}\right]. \quad (7)$$

The thermal speed of the RBF w_{RBF} must be specified, as well as the number of basis functions N_{RBF}, as well as the central location, $\vec{v}_c$, of each of the ψ_i used in the interpolation.

The interpolating function is then given by

$$f^{RBF}(v_\perp, v_\parallel) = \sum_i^{N_{RBF}-1} w_i\psi_i(\zeta), \quad (8)$$

with $\zeta = \vec{v} - \vec{v}_c$. Determination of the weights w_i is again performed through SVD estimation for the pseudoinverse giving a least square fit of f^{RFB} to $f(v)$.

3 Data

These non-parametric approximations have been applied to a range of data from the Parker Solar Probe SWEAP/SPAN ion instrument. SPAN is an electrostatic analyzer with 2048 energy bins capable of analyzing the solar wind thermal and non-thermal properties through measurements of the full 3D ion distribution. Figure 1 shows a distribution from PSP/SWEAP/SPAN at 2020-01-30/04:09. Panels a-c show the respective biMaxwellian fit, Hermite polynomial approximation, and RBF approximation to the VDF. We use sixth order Hermite polynomials for both the perpendicular and parallel directions. For the RBF, we use centroids for the basis functions at the location of energy bins that have atleast $1/e$ of the max level of the ion distribution. For the interval shown, this was 60 points, each corresponding to one basis function.

4 Signatures of Wave-Particle Interaction

We have previously studied the signatures of quasilinear heating rates of ion-kinetic waves with the ion distribution functions [5]. These results have shown that the ion distributions in the near sun solar wind are capable of absorbing energy stored in ion cyclotron waves [2, 13]. Using quasilinear heating rates [12] it is possible to measure the heating associated with each representation of the distribution function. The quasilinear heating rate is given by

$$\mathscr{H} = \int \frac{m_p v^2}{2} \frac{\partial f_p(\mathbf{v})}{\partial t} d^3\mathbf{v} =$$
$$\frac{\pi e^2}{2m_p^2} \int_0^\infty dk_\parallel \frac{1}{v_\perp} \hat{G}_k v_\perp \delta(\omega_k - k_\parallel v_\parallel - \Omega_p) \frac{\omega_k^2}{k_\parallel^2 c^2} I(k_\parallel) \hat{G}_k f_p(\mathbf{v}) d^3\mathbf{v} \tag{9}$$

[12, 5]. The spectrum of cyclotron waves I_k is determined following [2, 13]. Figure 2 shows the differential heating rates computed in 1km/s by 1km/s discrete bins in phase space. The total heating rate for each representation is 3.3×10^{-14} W/m^3 for the bi-Maxwellian, 3.0×10^{-15} W/m^3 for the Hermite Polynomial, and 8.3×10^{-15} W/m^{3}for the RBF approximation.

From the panels in Figure 2, it is clear that much of the heating occurs at speeds that are above the parallel and perpendicular thermals speeds (60 km/s and 50 km/s). This is reflected in the fact that the heating rates of the Hermite and RBF represnetaions are significantly higer. This highlights that capturing the non-thermal portion of the distribution is highly important in understanding the wave particle interactions occurring in the plasma. The thermal, or near-thermal approximations cannot capture the higher order structure present in the plasma [5]. The data-driven, non parametric approximations to these functions are significantly better at capturing the non-thermal portions of the distribution.

5 Discussion

Nonparametric and data-driven representation to particle velocity distributions are key in developing a greater understanding of wave-particle interactions. Accurate identification of plasma-wave dispersion relations [3, 6] depends on accounting for these non-thermal features.

Advancing machine learning methods to provide fast and accurate measurements of plasma distribution funcitons will significantly advance our understanding of kinetic processes in the near term, through e.g. providing physically accurate descriptions of plasmas. In the longer term, developing these methods into algorithms that can be implemented onboard a spacecraft will vastly improve in situ observations of plasmas through enabling higher fidelity measurements of full plasma distributions with relatively low number of degrees of freedom (we use 30-50 degrees of freedom to create the distributions shown here, out of 2048 total energy bins). Reducing the amount of data needed to capture fully nonthermal and kinetic effects will make measurements of physics occuring on wave-time scales possible.

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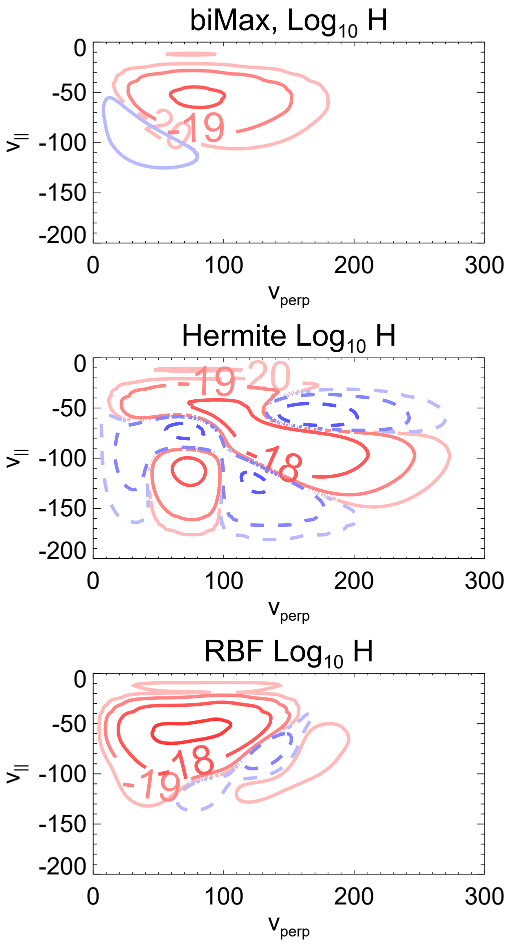

Figure 2. a) Differential heating rate for drifting bi-Maxwellian fit to distribution, red shows positive heating rates, corresponding to absorption of waves, blue shows regions of negative heating rates, corresponding to unstable regions of the distribution. b) Differential heating rate for Hermite polynomial approxiation to distribution. c) Differential heating rate for Radial Basis Function approximation. Data is gyrotropic, and thus the plot is limited to one half of the ion-cyclotron resonant region.

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