URSI GASS 2023, Sapporo, Japan, 19 – 26 August 2023

Significance of Kappa Distributed Electrons on Electrostatic Solitary Waves in Saturn's Magnetosphere

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Abstract

Over the past few decades, a range of theoretical models and experiments have provided ample evidence for the occurrence of kappa distributions in various space plasma environments, including the solar wind, planetary magnetospheres, the outer heliosphere, and the inner heliosheath. Among various planetary magnetospheres, Saturn's magnetosphere, for one, makes an excellent testing ground for the investigation of kappa-distributed electrons. Suprathermal electron populations may play a decisive role in the generation of plasma waves in Saturn's magnetosphere plasma. The Cassini spacecraft mission has detected Electrostatic Solitary Waves (ESWs) in Saturn's magnetosphere. Motivated by this fact and by observations of suprathermal electrons on Saturn, we have formulated a theoretical model to explore the significance of the electron parameters (density, temperature) in the evolution and the characteristics of ESWs occurring in Saturn's magnetosphere. Our method provides an efficient tool for understanding ESWs and their dependence on electron statistics, which may be vital in characterizing the microphysics of Saturn's magnetosphere.

1 Introduction

The presence of non-thermal particle distributions has been established by spacecraft measurements in the solar wind and in various space plasma environments. Such non-thermal populations are efficiently described by the Kappa distribution function, which is characterized by a long energy tail, resulting in a power law decrease for large arguments of the particle speed v, a feature that deviates from the Maxwellian behavior. Numerous theoretical studies have been carried out, motivated by observations of kappa distributions in various space plasma systems [1, 2, 3]. The magnetosphere of Saturn in particular makes for an ideal test-bed for the investigation of kappa distributed electrons [4]. Based on Cassini observations, Schippers [4] effectively examined the radial distribution of electron populations in Saturn's magnetosphere using an ad hoc two-kappa model for data fitting (in contrast with a bi-Maxwellian and with a kappa-Maxwellian model), thus establishing the coexistence of several kappa-distributed electron populations in the Saturnian environment. This coexistence of electron populations will serve as a crucial element in our study.

Suprathermal particles are known to affect the dynamics of electrostatic waves [5]. Instruments onboard the Cassini mission have detected Electrostatic Solitary Waves (ESWs) in Saturn's magnetosphere [1, 3]. ESWs are localized electrostatic structures, whose characteristic footprint is usually a symmetric bipolar pulse in the electric field component parallel to the magnetic field. An initial study of ESWs at Saturn was carried out by Williams et al., [1]. According to their observations, solitary waves tend to appear in the plasma boundary or in regions with rapid fluctuations in the magnetic field. They have reported pulse durations ranging from a few ms to a few tens ms, and peak-to-peak electric field amplitudes between a few mV/m and as high as 10 mV/m, in the vicinity of Saturn's magnetosphere. Later, Pickett et al. [3] carried out an extended survey of ESWs near Saturn. They obtained various types of ESWs, with amplitudes ranging from 100 μV/m to 140 μV/m, with pulse duration between several tens of microseconds to 250 μs. Motivated by these observations, we have established a bi-ion bi-electron fluid model to model ESWs. Adopting a pseudopotential (Sagdeev type) technique, we have established a pseudo-mechanical energy balance equation for the electrostatic potential, and have thus obtained nonlinear solutions numerically, in order to understand the salient features of ESWs in Saturn's magnetosphere.

2 Formulation

Saturn's magnetosphere is a typical example of a multi-component plasma system. The inner plasmasphere is predominantly composed of $O+$ and water group ions, whereas the outer plasmasphere is a mixture of $H+$, $O+$, and water-group ions. Despite being at a high altitude, the outer magnetosphere is made up of tenuous hot plasma that is dominated by $H+$ ions. Adopting a plasma model mimicking the rich plasma configuration found in Saturn's magnetosphere, we have considered an infinite, homogeneous, collisionless plasma comprising two types of ions (say, light and heavy ions) and two different electron populations (at different distributions, characterized by different temperatures). For simplicity in the analysis, both ions will be modeled as cold fluids. It is also assumed that two electron populations coexist at thermal equilibrium; these populations will be known as the "cool" and "hot" electron populations, based on their respective temperatures (Tc, Th, adopting a selfexplanatory notation).

The ion fluid-dynamical equations read

$$\frac{\partial n'_{ij}}{\partial t'} + \frac{\partial (n'_{ij} u'_{ij})}{\partial x'} = 0, \tag{1a}$$

$$\frac{\partial u'_{ij}}{\partial t'} + u'_{ij}\frac{\partial u'_{ij}}{\partial x'} = -\frac{z_{ij} e}{m_{ij}}\frac{\partial \phi'}{\partial x'}, \tag{1b}$$

where the double subscript "ij" ("i", for "ions"), denotes either the light ions (for $j = 1$) or the heavier ions (for $j = 2$). Therefore, n_{ij} and u_{ij} respectively denote the number density and the fluid speed of the corresponding (j^{th}) ion species. For the sake of generality, we have retained the charge multiplicity (state) of the ion species ($j = 1$ or 2) z_{ij} arbitrary. The elementary (electron) charge is obviously denoted by e, while m_{ij} is the mass of the ion species ($j = 1$ or 2). The electrons are assumed to obey a non-Maxwellian (kappa-type) distribution [2]. The electron density for the two electron populations, respectively denoted by the subscripts "c" (for cold) and "h" (for hot), are:

$$n'_{e,c} = n_{e,c,0}\left(1 - \frac{e\phi}{(\kappa_{ec} - \frac{3}{2})T_{ec}}\right)^{-\kappa_{ec}+\frac{1}{2}} \tag{2a}$$

$$n'_{e,h} = n_{e,h,0}\left(1 - \frac{e\phi}{(\kappa_{eh} - \frac{3}{2})T_{eh}}\right)^{-\kappa_{eh}+\frac{1}{2}} \tag{2b}$$

where $n_{e,c,0}$ and $n_{e,h,0}$ denote the corresponding equilibrium densities, and $T_{e,c}$ and $T_{e,h}$ are their corresponding temperature(s) (K_B denotes Boltzmann's constant, as usual). Here, $\kappa_{e,c}$ and $\kappa_{e,h}$ are the spectral indices corresponding to the cold and hot electrons, respectively. The Poission equation is given as,

$$\frac{\partial^2 \phi'}{\partial x^2} = -\frac{e}{\epsilon_0}\left[z_{i1} n'_{i,1} + z_{i2} n'_{i,2} - (n'_{e,c} + n'_{e,h})\right]. \tag{3}$$

Linear dispersion relation By linearizing the original fluid equations Eqs. (1) and (2) and assuming harmonic excitations $\sim \exp[i(kx - \omega t)]$ for all state variables, we obtain a linear dispersion relation in the form

$$\omega^2 = \omega^2_{peff}\frac{k^2}{k^2 + \lambda^{-2}_{Deff}} \tag{4}$$

where $\omega^2_{p,eff} = \omega^2_{p,i,1} + \omega^2_{p,i,2}$ is the sum of the plasma frequencies and

$$\begin{aligned} K^{-1}_{Deff} &= \lambda_{Deff} \\ &= \left\{\frac{e^2}{\epsilon_0}\left[\frac{n_{e,c,0}(\kappa_{e,c} - \frac{1}{2})}{K_B T_{e,c}(\kappa_{e,c} - \frac{3}{2})} + \frac{n_{e,h,0}(\kappa_{e,h} - \frac{1}{2})}{K_B T_{e,h}(\kappa_{e,h} - \frac{3}{2})}\right]\right\}^{-1/2} \end{aligned} \tag{5}$$

is the effective (Debye) screening length in our plasma model. From Eq. (4), the acoustic speed in our multi-component plasma model (C_s) can be obtained as

$$\begin{aligned} \lim_{k\to 0}\left(\frac{\omega}{k}\right) = C_s &= \left(\frac{\omega^2_{pi1} + \omega^2_{pi2}}{K^2_{D,eff}}\right)^{1/2} \\ &= \left(\frac{1 + \delta Q^2/\mu}{1 + Q\delta}\right)^{1/2}\left(\frac{z_{i1} K_B T_{eff}}{m_{i1}}\right)^{1/2}. \end{aligned} \tag{6}$$

where $T_{eff} = \left\{\frac{\zeta\left(\kappa_{e,c} - \frac{1}{2}\right)}{T_{e,c}\left(\kappa_{e,c} - \frac{3}{2}\right)} + \frac{\nu\left(\kappa_{e,h} - \frac{1}{2}\right)}{T_{e,h}\left(\kappa_{e,h} - \frac{3}{2}\right)}\right\}^{-1}$.

Nonlinear structures For this model, we derive the expression for the solitary wave solution by using Sagdeev pseudopotential technique, The basic formalism of Sagdeev pseudopotential is,

$$\frac{\partial^2 \Phi}{\partial \eta^2} = n_i - n_e = -\frac{\partial \Psi(\Phi)}{\partial \Phi}, \tag{7}$$

Since we anticipate stationary profile solutions, we may now express the fluid equations as scaled equations in a stationary frame by applying the transformation from $\{x,t\}$ to $\eta = x - Vt$, where V is the velocity of the localized structure. For that we have normalized Eqs. (1) -(3) by scaling over appropriate plasma quantities as follows. The time and space were scaled by the (light ion) plasma period $\omega^{-1}_{p,i,1} = \left(\frac{\epsilon_0 m_{i,1}}{n_{i10} z^2_{i1} e^2}\right)^{1/2}$ and by the characteristic length $\lambda_* = \left(\frac{\epsilon_0 K_B T_*}{n_{i,1,0} z_{i,1} e^2}\right)^{1/2}$, respectively, while the ion fluid speed variables were normalized by the (light ion) thermal speed $c_{i1} = (z_{i1} K_B T_*/m_{i1})^{1/2}$ (note that $c_{i1} = \omega_{p,i,1}\lambda_*$). The ion number densities ($n'_{i,1}$ and $n'_{i,2}$) were scaled by their corresponding equilibrium densities, i.e. $n_{i,1,0}$ and $n_{i,2,0}$. The electron densities were scaled by the (total) equilibrium electron density $n_{e,0}$. Finally, the electrostatic potential was scaled by $\frac{K_B T_*}{e}$. In the above quantities, we have defined the characteristic temperature (scale) $T_* = \frac{1}{1+Q\delta}\left\{\frac{\zeta\left(\kappa_{e,c} - \frac{1}{2}\right)}{T_{e,c}\left(\kappa_{e,c} - \frac{3}{2}\right)} + \frac{\nu\left(\kappa_{e,h} - \frac{1}{2}\right)}{T_{e,h}\left(\kappa_{e,h} - \frac{3}{2}\right)}\right\}^{-1}$, thus chosen so that the effect of both the temperatures and fractional densities of the two components would reflect on the analysis.

By adopting vanishing boundary conditions for the density and fluid speed variables, viz. $u_{ij} \to 0, n_{ij} \to 1$ (for $j = 1,2$) and also for the electrostatic potential $\Phi \to 0$ as $|x| \to \infty$, one finds the perturbed densities for the two ion species as

$$n_{i,1} = \left(1 - \frac{2\Phi}{V^2}\right)^{-1/2} \quad \text{and} \quad n_{i,2} = \delta\left(1 - \frac{2\Phi Q}{\mu V^2}\right)^{-1/2}. \tag{8}$$

Upon integration of Eq. 7, leads to a pseudo-energy balance relation in the form

$$\frac{1}{2}\left(\frac{d\Phi}{d\eta}\right)^2 + S(\Phi) = 0. \tag{9}$$

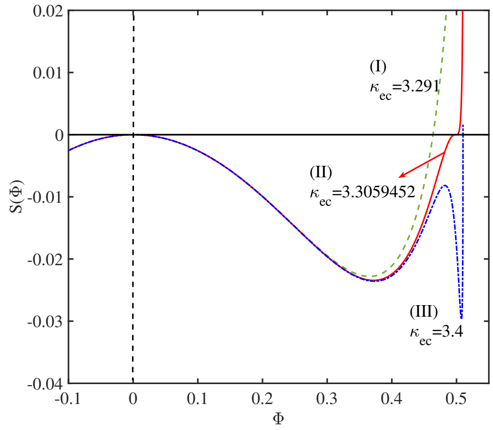

Figure 1. The Sagdeev pseudopotential corresponding to the value of $\zeta = 0.003291$ Curve (I) corresponds to RSW ($\kappa_{e,c} = 3.4$), Curve (II) corresponds to FTSW($\kappa_{e,c} = 3.3059452$), and Curve (III) ($\kappa_{e,c} = 3.291$)corresponds to SSW. Parameters are: $V = 1.01; Q = 16; z_{i1,2} = 1; \delta = 0.01; \beta = 0.30976; \kappa_{e,h} = 3.5$

where, the pseudopotential function $S(\Phi)$,

$$S(\Phi) = \left(1+Q\delta\right)^2\left(\zeta C_{e,c}+\nu\beta C_{e,h}\right)\times \left\{\zeta\left(1-\left(1-\frac{\Phi}{(1+Q\delta)(\zeta C_{e,c}+\nu\beta C_{e,h})(\kappa_{e,c}-\frac{3}{2})}\right)^{(-\kappa_{e,c}+\frac{3}{2})}\right) +\frac{\nu}{\beta}\left(1-\left(1-\frac{\beta\Phi}{(1+Q\delta)(\zeta C_{e,c}+\nu\beta C_{e,h})(\kappa_{e,h}-\frac{3}{2})}\right)^{(-\kappa_{e,h}+\frac{3}{2})}\right)\right\} +V^2\left(1-\left(1-\frac{2\Phi}{V^2}\right)^{1/2}\right)+\delta V^2\mu\left(1-\left(1-\frac{2Q\Phi}{\mu V^2}\right)^{1/2}\right).$$

We shall now focus on the existence of pulses of various types (i.e. bipolar, flat-top or monopolar profiles) and the possible transition between one type and another, focusing on how this is affected by the non-Maxwellian character of the electrons, from first principles. Inspired by evidence of a twofold-kappa electron configuration in Saturn's magnetosphere [4], we will focus on varying the relative concentration of the cold electron component (by varying the values of ζ) and the cold electron spectral index $\kappa_{e,c}$. In Saturn's magnetosphere the plasma is mixture of O^+ and H^+ ions along with the water group ions. Hence the ion parameters, $\mu = 16$, and $Q = 1$,and keeping all other parameters fixed as: pulse velocity $V = 1.01$, electron temperature ratio $\beta = 0.294$, $\zeta = 0.003291$ and spectral index $\kappa_{e,h} = 3.5$.

Varying the value of $\kappa_{e,c}$ for the cold electron population, we have thus obtained an indicative class of nonlinear coherent structures, as depicted in Fig. 1. In Fig. 1, the leftmost curve (I) ($\kappa_{e,c} = 3.4$) corresponds to a regular solitary wave (RSW), whereas the middle curve (II) ($\kappa_{e,c} = 3.3059452$) represents a Flat Top solitary wave and curve III ($\kappa_{e,c} = 3.291$) represents a supersolitary wave (SSW).

To explore the physical characteristics of the solitary wave

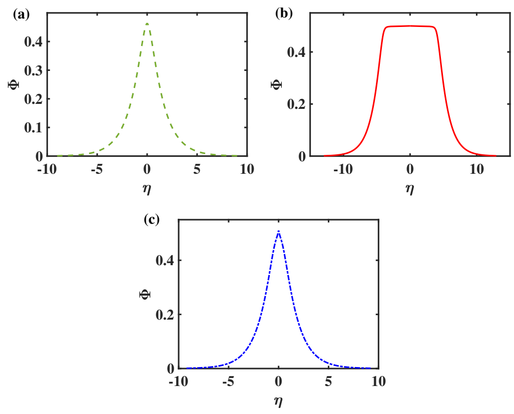

Figure 2. Electrostatic potential profiles corresponding to different $\kappa_{e,c}$ values; the depicted curves represent (I) RSW, for $\kappa_{e,c} = 3.4$, Curve (II) corresponds to FTSW($\kappa_{e,c} = 3.3059452$), and Curve (III) ($\kappa_{e,c} = 3.291$)corresponds to SSW. Here the parameter values are: $V = 1.01; Q = 16; z_{i1,2} = 1; \delta = 0.01; \beta = 0.294; \kappa_{e,h} = 3.5$.

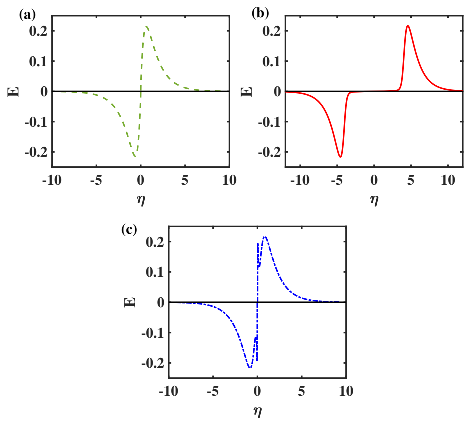

Figure 3. Electric field waveforms corresponding to different $\kappa_{e,c}$ values; the depicted curves represent (I) RSW, for $\kappa_{e,c} = 3.4$, Curve (II) corresponds to FTSW($\kappa_{e,c} = 3.3059452$), and Curve (III) ($\kappa_{e,c} = 3.291$)corresponds to SSW. Here the parameter values are: $V = 1.01; Q = 16; z_{i1,2} = 1; \delta = 0.01; \beta = 0.294; \kappa_{e,h} = 3.5$.

solutions depicted in Fig. 1, in Fig. 2 and in Fig. 3, we have plotted the corresponding potential and Electric field profiles, respectively. Figures 2(a) and 3(a) represents the usual bell shaped potential profiles and bipolar E-field pulse correspond to a RSW, respectively, while Fig. 2(c) and 3(c) represents the usual wiggled bell shaped potential profiles and wiggled bipolar E-field pulse correpsond to an SSW (curve III in Fig. 1), respectively. The morphology of the structure represented as in curve II of Fig. 1, however, is different from that of a conventional one as is evident from the associated electric field (Fig. 3(c)) and potential profiles (Fig. 2(c)). The potential profile show a a flat top profile which is the signature of a flat-top solitary waves (FTSW), and in the electric field profile the distances between the two peaks are relatively large, compared to the characteristic width of the each peak and also in relation with the standard bipolar forms [6].

Validation of the model Following the observational data presented by Schippers et al., [4] we have chosen the electron parameters in Saturn's inner magnetosphere as, $T_{e,c} = 23.6eV$, $T_{e,h} = 2400eV$, $n_{e,c} = 5.80cc$, $n_{e,h} = 0.32cc$, and $\kappa_{e,h} = 4.24$. The chosen set of parameters gives the, electron temperature ratio $\beta = 0.00983$, cold electron concentration $\zeta = 0.9477$, and keeping $\kappa_{ec} = 3.5$. From these values we have estimated the peak to peak amplitude of the solitary wave pulse, $E_{pp} = 47.4mV/m$. According to Pickett et al., [3] the peak to peak amplitude of the observed solitary wave pulses in Saturn magnetosphere ($10R_s$) is between $100\mu V/m$ and $140mV/m$ approximately. Hence, the estimated E_{pp} value was found to be consistent with Cassini observations.

3 Conclusion

In this work we have analysed the dynamics of electrostatic solitary waves in a multi-component plasma, where velocity distribution of electrons are modelled using kappa (κ) distribution. Adopting an analytical (non-perturbative) technique, we have explored the significance of the non-Maxwellian nature of the super-thermal electron population(s) in the formation and characteristics of various types of solitary waves in Saturn Magnetosphere plasma. From our analysis it is obvious that the spectral index κ plays a decisive role in the evolution of solitary structures. As the spectral index κ (either κ_{ec} or κ_{eh}) decreases in value, regular solitary waves give their place to large amplitude super-nonlinear structures (either Super-solitary Waves or Flat-top Solitary Waves). A smaller value of $\kappa_{e,c}$ enhances the suprathermal (cold) electron population: this means that, in Saturn's magnetosphere, an increase in the nonthermal cold electron component results in the formation of larger amplitude solitary structures. Our model has been validated with real observations of Electrostatic Solitary Waves in Saturn's magnetoshere [3], and the theoretically predicted values came out to match the characteristics of the observed ESWs to a highly satisfactory extent.

Acknowledgements

Authors SSV and IK gratefully acknowledge financial support from Khalifa University's Space and Planetary Science Center (Abu Dhabi, UAE) under Grant no. KU-SPSC-8474000336. IK acknowledges support from KU via the CIRA (Competitive Internal Research Award) CIRA-2021-064 (8474000412) and FSU (Faculty Start-Up award) FSU-2021-012 (8474000352) projects.

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